OPTIMAL PRODUCTION AND MAINTENANCE PLANNING FOR RANDOM DEMAND WITH VARIABLE PRODUCTION RATE AND SUBCONTRACTING CONSTRAINTS

[1]AYED Souheil, [1]DELLAGI Sofiene*, [1]REZG Nidhal
INRIA Nancy Grand Est /LGIPM – ENIM , 1 route d'Ars Laquenexy. CS65820, 57078 Metz Cedex 3 FRANCE

ABSTRACT

This paper deals with an industrial problematic of a manufacturing system called to satisfy a random demand during a finite horizon with a given service level. The manufacturing system production rate is limited. To respond to this demand, the manufacturing calls upon subcontracting. In order to minimize the inventory and the production cost, the manufacturing system must operate with a variable production rate. The manufacturing system M1 is subject to a random failure. We assume that, the deterioration of the machine M1 depends both on time and production rate. The object of this study is to determine an optimal production plan taking into account the machine deterioration following its production rate. The subcontractor manufacturing system M2 is out of control. To start with, under a given service level with a subcontracting constraint, we establish a production plan minimizing the inventory and production cost. Thereafter, starting with the previous production plan, we derive an optimal production plan taking into account the degradation of the machine, minimizing simultaneously: the production, the inventory and the degradation cost. In a next stage, we also propose another optimal production plan by minimizing costs of production, inventory and maintenance. In this new optimal production plan, we introduce a preventive maintenance plan, taking into account the production rate variation. Two numerical examples are presented to illustrate the two proposed approach.

KEYWORDS

Integrated maintenance, variable production rate, degradation, production plan, subcontracting.

INTRODUCTION

The integrated maintenance has been the subject of several studies in recent years. It has been proven that the maintenance management is closely linked to both, production structure and demand nature. Buzacott. (6) among the first authors who treated the problem of maintenance and production, he studied the role of buffer stock on increasing the system productivity. In the JIT context, Abdelnour et al. (1), and Chan et al. (7) proposed a simulation model to evaluate the performance of a production line operating in push system. Van Brachte (16) proposed a preventive maintenance policy considering the machine age and the stock capacity between two machines.

Concerning subcontracting, it has grown in the industrial world in virtually all domains as noted by Amesse et al. (2). This practice is not always justified by production costs. It is part of cooperation logic and coordination based on technological incentives, to satisfy customers in terms of quantity and delay. The above was treated by Andersen. (3) and Bertrand et al. (5). Recently, in the context of integrated maintenance, Dellagi. et al. (9) developed a maintenance strategy integrating a subcontracting constraint. They treated a production system represented by a machine producing a single product type to satisfy a constant demand during time. The machine calls upon the sub-contracting represented by a second machine to complete the entire demand exceeding the maximum machine capacity. Following the results obtained by Dellagi et al. (9), the authors Dellagi et al. (10) continued in the same context to address the problem but with two subcontractors. They defined a policy of switching between sub-contractors. The optimization of this strategy is sequential. It consists in determining first, the optimal age of preventive maintenance, secondly calculating the optimal switching date. For their part, Cormier et al. (8) proposed an analytical model to optimize maintenance and production with subcontracting constraint, by integrating a shortage stock level caused by machine downtime for both contractor and subcontractor. In the cited articles treating the sub-contracting, demand is assumed known, constant and within an infinite horizon. Whereas in our study, the demand is random on a finite time horizon. To meet such a demand while minimizing production and inventory costs, it is necessary to vary the production rate. In reality, the failure rate increases with time and according to the use of the equipment. It is obvious, when we produce more, we degrade more the machine. Moreover, a change in production rate can also be beneficial to reach production goals when unpredicted events happen in the system that disturbs the original production plan. Khouja and Mehrez. (13) were the first to consider a variable production rate in the classical economic production quantity (EPQ) model. In their work, they assumed that product quality depends on the production rate. In the literature, the consideration of the equipment failure according to the production rate is rarely studied. Among these works, we can cite Hu et al. (12) who discussed the conditions of optimality of the hedging point policy for production systems in which the failure rate of machines depends on the production rate. Others like Liberopoulos and Caramanis. (14) studied the optimal flow control of single-part-type production systems with homogeneous Markovian machine failure rates dependant on production rate. In all these cited studies above, when treating the failure problem dependency on production rates, they assumed that the law of failure is exponentially distributed.

The optimization of simultaneous maintenance production is a complex task given the various uncertainties associated with the decision process. These uncertainties are usually due to the randomness of the demand, causing the incapacity or predicting the demand behaviour throughout future periods. Silva and Cezarino. (15) dealt with a chance-constrained stochastic production-planning problem under the hypotheses of imperfect inventory information variables and by computing the expected value of the cost.

More recently Hajej et al. (11) dealt with combined production and maintenance plans for a manufacturing system satisfying a random demand over a finite horizon. In their model, they assumed that the failure rate depends on the time and the production rate. In our study, we build on Hajej's et al (11) model. The given manufacturing system cannot ensure the total demand over the horizon, it calls upon the subcontracting. The manufacturing system is subject to random failures. The failure rate depends on the time and the production rate which is variable over the production horizon. The first approach is to establish a production plan optimizing the production and inventory cost with a given service level. To solve this, we formulated an inventory and production problem as a constrained stochastic linear quadratic problem generalizing the HMMS (Holt, Modigliani, Muth and Simon) model. In the next stage, the previous production plan is then used to derive the cost of degradation using a proposed degradation unit cost. The optimal production plan is obtained by minimizing the production, the inventory and the degradation cost.

This paper is organized as follows. The next section describes the problem, the used notation and the adopted production policy. In section 3, the mathematical model is presented expressing the total expected cost determining the production plan. The section 4, present the production plan influence on the manufacturing system degradation. This influence is taken into account in the determination of the optimal production plan to minimize the production, the inventory and the degradation cost. The fifth section is dedicated to the numerical example to show the proposed approach efficiency. Finally, a summary of the work together with indications about extensions currently under consideration is provided in the last Section of the paper.

PROBLEM STATEMENT

General Problem Description

In this study, we deal with an industrial problematic of manufacturing enterprise producing one part-type through a single operation in order to satisfy a random demand over a finite horizon *H*. This demand is characterized by a normal distribution with an average demand $\hat{d}$ and a standard deviation *σ*. The maximum production rate of this manufacture U_1^{max} is lower than the average demand $\hat{d}$ and its unit production cost is *Cpr1*. In order to satisfy this random demand with a given inventory service level *α*, and to avoid shortage due to the manufacturing system unavailability, the enterprise has to build a stock. That's why, it calls upon another production enterprise, called subcontractor. The unsatisfied demands are lost and induce a demand lost cost. The process machine M1 of the manufacturing enterprise is subject to a random failure. The probability density function of time to failure is *f(t)*, while the failure rate *λ(t)* is increasing in both time and production rate *u(t)*. In another step, we integrate a preventive maintenance policy optimizing simultaneously production, inventory and maintenance costs.
The study of this problematic is achieved in two steps. In the first, we establish a production plan minimizing production and inventory cost under service level constraint. Thereafter, we analyze the influence of the production rate variance on the degradation of the manufacturing system during the production horizon. To evaluate this influence, we propose a unit degradation cost. In the second step, we implement a preventive maintenance policy. Using the production plan established in the first step, we propose a new optimal plan, minimizing simultaneously production, inventory and maintenance costs.
The subcontractor's manufacturing system M2 maintenance is out of control. The only information about its maintenance is the availability rate *ß2*. The availability rate is defined by the satisfied demand number divided by the total number demand in a constant period. The machine M2 is characterized too by its maximal production rate U_2^{max} and its unit production cost *Cpr2*. The industrial problem is illustrated in the figure 1.

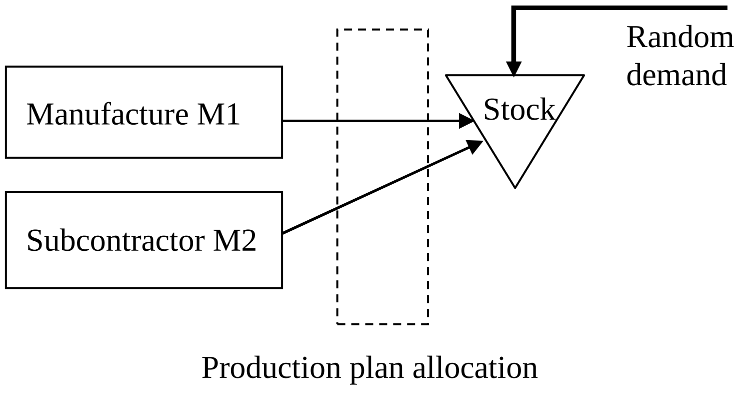

Fig. 1: Industrial problem

Notation

H : finite production horizon
Δt: period length of production
S_k: inventory level at the end of the period k (k=1,…….,$H/\Delta t$)
$U_{i,k}$: production level at period k of machine Mi, i∈{1,2}
d_k : demand quantity at period k
$Cpri$: unit production cost of machine Mi, i∈{1,2}
Cs: holding cost of a product unit during the period k
$R(t)$: reliability function
CM: maintenance cost
C_{pm}: preventive maintenance action cost
C_{cm}: corrective maintenance action cost
mu: monetary unit
U_i^{max}: maximal production rate of machine Mi, i∈{1,2}
α: probabilistic index (related to customer satisfaction and expressing the service level)

PRODUCTION STRATEGY

This study concerns a single stochastic inventory balance system. The object is to optimize the expected production and inventory costs over a finite horizon H. It is assumed that the horizon H is partitioned equally into H periods of length Δt. The demand is satisfied at the end of each period. In our analytical model we assume that: inventory and production costs $Cpri$, Cs, the demand standard deviation σ and the demand average $\hat{d}$ are known and constant.
To establish the production plan, the first step consist to determine U_k which represent the quantity to produce by both machine M1 and M2 for each period k. in the second step, we allow to each machine the quantity to produce.

PRODUCTION PLAN

We recall that, our objective in this part is to determine the production plan over a time horizon H, minimizing the expected production and inventory costs. Thus, this kind of problem can be formulated as a linear-stochastic optimal control problem under threshold stock level constraint, with production rate as variable decision. We suppose that $\{f_k,\ k=1,2,...,H\}$ represent inventory and production costs, and $E\{\}$ denotes the mathematical expectation operator. Referring to Hajej's et al (11), we formulate our problem as following :

$$\underset{U(k)}{Min}\left(E\left\{\sum_{k=0}^{H-1} f_k\left(S_k,U_k\right)+f_H\left(S_H\right)\right\}\right) \tag{1}$$

U_k represent the production of both machines M1 and M2, with : $U_k = U_{1,k} + U_{2,k}$.
We note that the expected cost at the period H does not depend on the production rate U_k , because the demand is satisfied at the end of each period. Then, in this period, we consider only inventory cost.
The inventory balance equations for each time period is formulated in this way:

$$S_{k+1} = S_k + U_{1,k} + U_{2,k} - d_k \tag{2}$$

To prevent shortage, the service level requirement constraint for each period as well as a lower bound on inventory variables is as follow:

$$\text{Prob}\left[S_{k+1} \geq 0\right] \geq \alpha \text{ with } k \in \{1, 2, ..., H-1\} \quad (3)$$

The constraint defining an upper bound on the production level during each period k is:

$$0 \leq U_k \leq U_{\max}^1 + U_{\max}^2 \text{ with } k \in \{1, 2, ..., H-1\} \quad (4)$$

In our model, we use quadratic costs to penalize both excess and shortage of inventory (HMMS model). The quadratic total expected cost of production and inventory over the finite horizon H can then be expressed as follow:

$$F(u) = \sum_{k=0}^{N} f_k\left(u_{i,k}, S_k\right) = C_s E\left\{S_H^2\right\} + \sum_{k=0}^{H-1}\left[C_s E\left\{S_k^2\right\} + C_{pr1}U_{1,k}^2 + C_{pr2}U_{2,k}^2\right] \text{ with } k \in \{1, 2, ..., H-1\} \quad (5)$$

To facilitate the resolution of our complex problem due to the stochastic demand and inventory, we transform our problem into an equivalent deterministic one which will be then easier to solve.

$$F(u) = C_s \hat{S}_H^2 + \sum_{k=0}^{H-1}\left[C_s \hat{S}_k^2 + C_{pr1}U_{1,k}^2 + C_{pr2}U_{2,k}^2\right] + C_s(\sigma_d)^2 \frac{H(H+1)}{2}$$

Proof:

For $d_k = \hat{d}_k$, the inventory balance equation becomes:

$$\hat{S}_{k+1} = \hat{S}_k + \hat{U}_k - \hat{d}_k$$

Seeing that U_k is constant for each interval Δt, we have $\hat{U}_k = U_k$ and $Var_{U_k} = 0$

The inventory variable S_k is statistically described by its mean; $E\{S_k\} = \hat{S}_k$ and its variance Var_{S_k} $Var_{S_k} = E\left\{(S_k - \hat{S}_k)^2\right\}$

The balance equation (2) can be reformulated in this way:

$E\{S_{k+1}\} = E\{S_k\} + U_{1,k} + U_{2,k} - d_k$ this allow writing:

$$\hat{S}_{k+1} = \hat{S}_k + U_{1,k} + U_{2,k} - d_k \quad (6)$$

If we make the difference between equation (2) and (6) we obtain:

$$S_{k+1} - \hat{S}_{k+1} = S_k - \hat{S}_k - (d_k - \hat{d}_k)$$
$$\Rightarrow (S_{k+1} - \hat{S}_{k+1})^2 = ((S_k - \hat{S}_k) - (d_k - \hat{d}_k))^2$$
$$\Rightarrow E\left\{(S_{k+1} - \hat{S}_{k+1})^2\right\} = E\left\{((S_k - \hat{S}_k) - (d_k - \hat{d}_k))^2\right\}$$
$$\Rightarrow E\left\{(S_{k+1} - \hat{S}_{k+1})^2\right\} = E\left\{(S_k - \hat{S}_k)^2\right\} + E\left\{(d_k - \hat{d}_k)^2\right\} - 2E\left\{(S_k - \hat{S}_k)(d_k - \hat{d}_k)\right\}$$

Since S_k and d_k are independent random variables we can deduce that:

$$E\left\{(S_k - \hat{S}_k)(d_k - \hat{d}_k)\right\} = E\left\{(S_k - \hat{S}_k)\right\} E\left\{(d_k - \hat{d}_k)\right\}$$

Exploiting the linearity of the expectation we can write:

$$E\left\{(S_k - \hat{S}_k)\right\} = E\{S_k\} - E\left\{\hat{S}_k\right\} = 0 \text{ And } E\left\{(d_k - \hat{d}_k)\right\} = E\{d_k\} - E\left\{\hat{d}_k\right\} = 0$$

Therefore

$E\left\{(S_{k+1} - \hat{S}_{k+1})^2\right\} = E\left\{(S_k - \hat{S}_k)^2\right\} + E\left\{(d_k - \hat{d}_k)^2\right\}$ Consequently

$$(\sigma_{S_{k+1}})^2 = (\sigma_{S_k})^2 + (\sigma_{d_k})^2$$

If we assume that $\sigma_{S_0} = 0$ and σ_{d_k} is constant and equal to σ_d for all k's, we can deduce that:

$$(\sigma_{S_k})^2 = k(\sigma_{d_k})^2$$

Since $Var_{S_k} = E\left\{(S_k - \hat{S}_k)^2\right\} = E\left\{S_k^{\ 2}\right\} - \hat{S}_k^2$

And $Var_{S_k} = (\sigma_{S_k})^2 = k(\sigma_{d_k})^2$

We can write $E\left\{S_k{}^2\right\} - \hat{S}_k^2 = k(\sigma_d)^2$

$$E\left\{S_k{}^2\right\} = k(\sigma_d)^2 + \hat{S}_k^2 \quad (7)$$

$\Rightarrow E\left\{S_k{}^2\right\} = k(\sigma_d)^2 + \hat{S}_k^2$

Substituting (7) in the expected cost (5) we obtain:

$$F(u) = C_s\,\hat{S}_H^2 + \sum_{k=0}^{H-1}\left[C_s\hat{S}_k^2 + C_{pr1}U_{1,k}^2 + C_{pr2}U_{2,k}^2\right] + C_s(\sigma_d)^2\sum_{k=0}^{H-1}k$$

$$F(u) = C_s\,\hat{S}_H^2 + \sum_{k=0}^{H-1}\left[C_s\hat{S}_k^2 + C_{pr1}U_{1,k}^2 + C_{pr2}U_{2,k}^2\right] + C_s(\sigma_d)^2\,\frac{H(H+1)}{2} \quad (8)$$

THE SERVICE LEVEL CONSTRAINT

In this part, we introduce the service level constraint. To continue transforming the problem into a deterministic equivalent, we consider a service level constraint in a deterministic form specifying certain minimum cumulative production quantities depending on the service level requirements.

Lemma

We recall that, α defines the service level constraint. This constraint is expressed as follow: $\text{Prob}\left[S_{k+1} \ge 0\right] \ge \alpha$ with $0 \le U_k \le U_{\max}^1 + U_{\max}^2$ then, for *k=1,2 ,..,H-1* we determine:

$$U_k \ge U_\alpha(S_k, \alpha)\ ;\ U_k = U_{1k} + U_{2k} \quad (9)$$

Where:

$U_\alpha()$: Minimum cumulative production quantity and

$U_\alpha(S_k,\alpha) = V_{d_k}\,\varphi_{d_k}^{-1}(\alpha) + \hat{d}_k - S_k \quad k = 0,1,\dots,H-1$, With:

V_{d_k} : Variance of demand d at period k.

φ_{d_k} : Cumulative Gaussian distribution function with $\hat{d}_k$ mean and finite variance $V_{d_k} \ge 0$.

$\varphi_{d_k}^{-1}$: Inverse distribution function.

Proof of lemma:

$\text{Prob}\left[S_{k+1} \ge 0\right] \ge \alpha$ with $0 \le U_k \le U_{\max}^1 + U_{\max}^2$

$\Rightarrow \text{Prob}\left[S_k + U_k - d_k \ge 0\right] \ge \alpha$

$\Rightarrow \text{Prob}\left[S_k + U_k \ge d_k\right] \ge \alpha$

$\Rightarrow \text{Prob}\left[S_k + U_k - \hat{d}_k \ge d_k - \hat{d}_k\right] \ge \alpha$

$$\Rightarrow \text{Prob}\left[\frac{S_k + U_k - \hat{d}_k}{V_{d_k}} \ge \frac{d_k - \hat{d}_k}{V_{d_k}}\right] \ge \alpha \quad (10)$$

This equation is of $\text{Prob}\left[Y \geq X\right] \geq \alpha$ the form of, with $X = \dfrac{d_k - \hat{d}_k}{V_{d_k}}$ is a Gaussian random variable representative of the demand d_k, and φ_{d_k} is a cumulative Gaussian distribution function of the form $F(Y) \geq \alpha$ such us:

$$\varphi_{d_k}\left(\frac{S_k + U_k - \hat{d}_k}{V_{d_k}}\right) \geq \alpha \tag{11}$$

Since $\lim\limits_{d_k \to -\infty} \varphi_{d_k} = 0$ and $\lim\limits_{d_k \to +\infty} \varphi_{d_k} = 1$, the function φ_{d_k} is strictly increasing, and we note that is indefinitely differentiable. That's why we conclude that is φ_{d_k} is invertible, thus:

$$\frac{S_k + U_k - \hat{d}_k}{V_{d_k}} \geq \varphi_{d_k}^{-1}(\alpha)$$

$$\Rightarrow S_k + U_k - \hat{d}_k \geq \varphi_{d_k}^{-1}(\alpha) V_{d_k}$$

$$\Rightarrow U_k \geq \varphi_{d_k}^{-1}(\alpha) V_{d_k} - S_k + \hat{d}_k \tag{12}$$

We can conclude that $U_\alpha\left(S_k, \alpha\right) = V_{d_k}\ \varphi_{d_k}^{-1}(\alpha) + \hat{d}_k - S_k$ with $k = 0,1,\ldots,H-1$

Using the last lemma and equation(8), we resume the equivalent deterministic model as follows:

$$\underset{U}{Min} = C_s \hat{S}_H^2 + \sum_{k=0}^{H-1}\left[C_s \hat{S}_k^2 + C_{pr1} U_{1,k}^2 + C_{pr2} U_{2,k}^2\right] + C_s (\sigma_d)^2 \frac{H(H+1)}{2}$$

Subjected to:

$$S_{k+1} = S_k + U_{1k} + U_{2k} - d_k$$

$$U_{1,k} + U_{2,k} \geq \varphi_{d_k}^{-1}(\alpha) V_{d_k} - S_k + \hat{d}_k \text{ with } k = 0,1,\ldots,H-1 \text{ and } 0 \leq U_{1,k} + U_{2,k} \leq U_1^{\max} + U_2^{\max}$$

The production plan gives us the quantity U_k to produce by both machines M1 and M2 for each period k. If this value is less than the maximum production rate of the machine M1, M1 produces all the quantity. If not, the machine M1 produce with its maximum production rate, and the machine M2 produce the rest. See Figure 2.

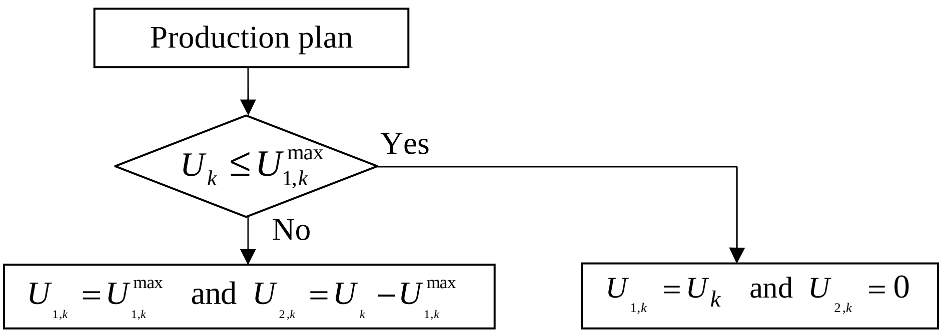

Fig. 2: production plan dispatching

INFLUENCE OF THE MACHINE DEGRADATION ON THE PRODUCTION PLAN

DEGRADATION COST ESTIMATION

In this section, we want to prove the effect of the machine degradation on the production plan previously established. We recall that in our model we assume that the failure rate $\lambda(t)$ is increasing in both time and production rate $U(t)$. Since the machine production rate is variable during the horizon *H*, the degradation will be variable too. To estimate the influence of the degradation on the production plan, we adopt a unitary degradation cost C_λ. The total cost degradation is obtained by multiplying the unitary degradation cost C_λ by the failure rate during the horizon *H*. Considering that the failure rate is continue and cumulative, the final failure rate in the end of the horizon *H* is the sum of the failure rate of each period.

ANALYTICAL STUDY

Each period *k* of the horizon *H* is characterized by its own production rate U_k established from the production plan. The failure rate evolves in each interval according to the production rate adopted in this interval. It also depends on the failure rate cumulated at the end of the previous period. As per Hajej's et al (11) approach, the degradation in the end of the period is then accounted for. In fact, the failure rate in the interval *k* is expressed as following:

$$\lambda_k(t) = \lambda_{k-1}(\Delta t) + \frac{U_k}{U_{\max}} \lambda_n(t) \tag{13}$$

With $\lambda_{k=0} = \lambda_0$ and $\Delta \lambda_k(t) = \frac{U_k}{U_{\max}} \lambda_n(t)$

$\lambda_n(t)$ is the nominal failure rate corresponding to the maximal production rate.

We recall that Hajej's et al (11) assumed that machine degradation is linear according to the production rate.

We can write the failure rate function like this:

$$\lambda_k(t) = \lambda_0 + \sum_{l=1}^{k-1} \frac{U_l}{U_{\max}} \lambda_n(\Delta t) + \frac{U_k}{U_{\max}} \lambda_n(t) \ \textit{with} \ t \in [0, \Delta t] \tag{14}$$

We note that $\lambda_n(\Delta t)$ is the degradation at the end of the production period (Δt).

The degradation penalizing cost is equal to:

$$C_\lambda \sum_{k=0}^{H} \lambda_k^2 \tag{15}$$

From equation (8) and (15) we obtain the total cost including the production, the inventory and the degradation cost:

$$CT(U) = C_s \hat{S}_N^2 + \sum_{k=0}^{N-1} \left[C_s \hat{S}_k^2 + C_{pr1} U_{1,k}^2 + C_{pr2} U_{2,k}^2 \right] + C_s (\sigma_d)^2 \frac{H(H-1)}{2} + C_\lambda \sum_{k=0}^{H} \lambda_k^2 \tag{16}$$

OPTIMISATION

Taking the degradation cost into account, we then optimise the production plan established previously by minimizing the total cost, which include: production, inventory, shortage and degradation.

The key of this optimization strategy is in the minimization of the total cost including production, inventory and the degradation cost, by transferring a production from a machine M1 to the subcontractor machine M2 as shown in the following figure:

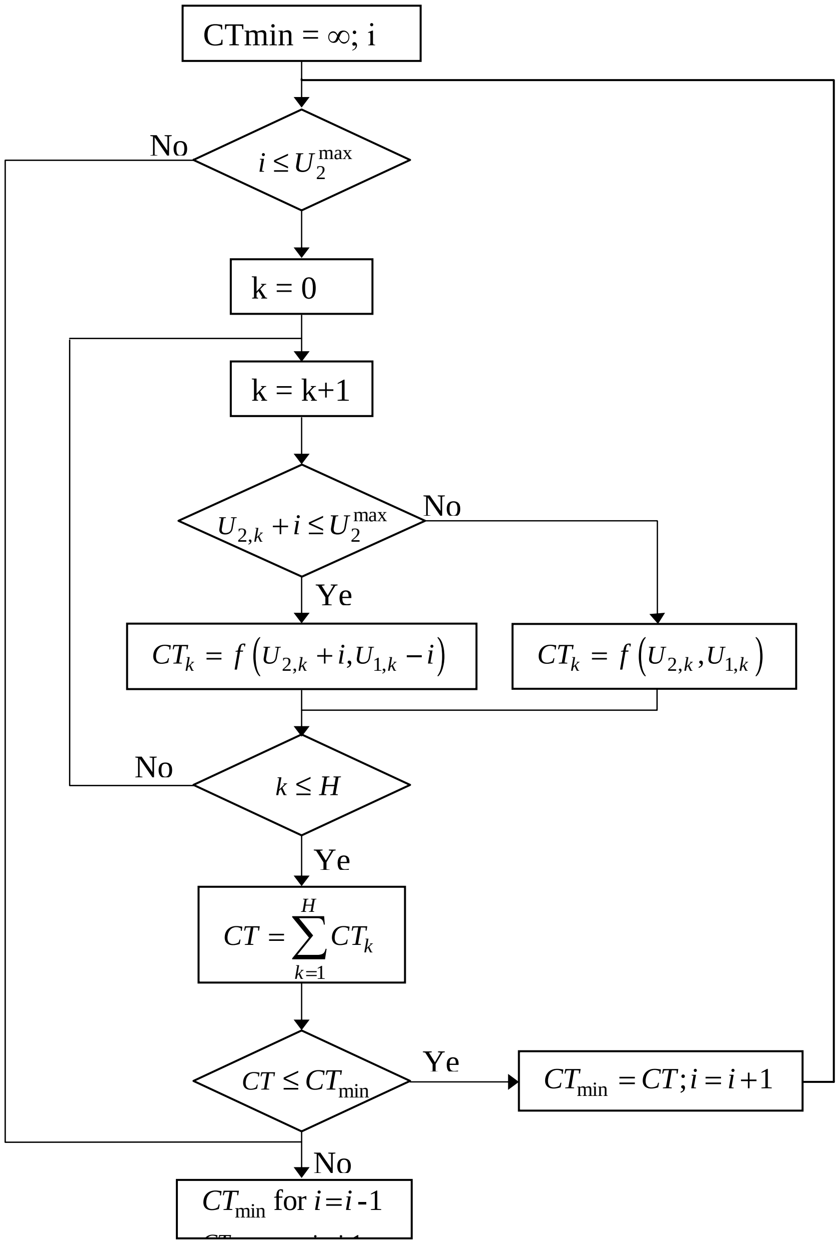

Fig. 3: production plan optimization with degradation cost.

NUMERICAL EXAMPLE

In order to illustrate the model developed previously, we consider a company represented by machine M1 which has to satisfy a stochastic demand assumed Gaussian over a finite horizon

H; with a mean $\hat{d}_k$, variance V_{d_k} and number of period *H* equal to *120*, which is extracted from a historical sales report. To satisfy the demand with a given service level α, the company calls up to a subcontractor represented by a machine M2. The machine M1 has a degradation law characterized by a Weibull distribution. The Weibull scale and shape parameters are *ß*=100 and γ=2. The only information known about M2 reliability is the service level *ß2* which represent its availability. The following data are used for the other parameters: *Cpr1*=7*mu*, *Cpr2*=25*mu*, U_1^{max}=11, U_2^{max}=8, *ß2*=0.93, service level *α*=0.9, *Cs*=0.65*mu*, initial inventory S_0=15, degradation cost *Cλ*=35*mu*. The expected demand $\hat{d}_k = 15$ and the variance $V_{d_k} = 1.21$. The mean demand is presented in the table 1.

15	17	15	15	15	14	16	14	16	15	15	15	15	15	15	13	15	15	16	13	15	15	14	16
16	16	14	15	15	14	15	16	14	16	14	14	17	16	14	14	15	15	15	14	15	14	14	15
14	14	15	13	15	15	17	14	16	16	15	14	14	13	18	15	14	13	13	16	15	15	14	14
15	15	14	14	13	12	16	16	15	15	15	16	14	17	16	16	15	16	13	14	16	14	14	16
16	13	17	14	17	14	16	14	16	16	14	15	14	14	15	15	16	14	16	14	15	15	14	14

Table 1: Mean demand

The production plan corresponding to the previous demand for respectively M1 and M2 is as following (table 2)

19	21	14	14	15	12	18	12	18	13	15	14	14	14	15	10	17	15	17	9	17	15	12	18
17	16	12	15	14	12	15	18	11	18	12	13	20	17	11	13	15	15	14	12	16	12	13	15
13	13	15	10	17	15	19	11	18	16	14	12	13	11	24	17	12	12	13	20	14	15	12	13
15	15	12	13	11	11	21	19	14	14	14	17	12	20	16	17	13	17	10	15	18	12	13	18
17	9	22	14	20	11	18	12	18	17	11	15	12	14	15	14	17	12	18	12	15	14	12	13

Table 2: Production plan

To optimize the production plan obtained considering the degradation cost, we calculate first the degradation cost relative to the production plan and we add it to the total cost. Then, we decrease the amount that the machine M1 has to produce by a unit, and we increase that of the machine M2, and we calculate again the total cost. We repeat this procedure until finding the optimal cost as showing at the figure 3. The results of the optimization are presented in the table number 3.

Case (+i,-i)	Degradation cost	Total cost	Service level %
0	120847,98	304998,87	100
(-1,+1)	101609,65	286489,42	100
(-2,+2)	86150,62	272279,36	100
(-3,+3)	73751,38	266552,14	98,33
(-4,+4)	64722,94	265454,38	77,5

Table 3: The production plan optimization result with degradation cost.

The table first column represents the quantity case transferred from the machine M1 to the machine M2. We note that, the minimum total cost is reached at the (-4, +4) case with a service level equal to 77.8%. This means that, if we decrease the machine M2 production plan of 4 unity and we increase that machine M1 of 4, we obtain the minimal total cost. But, the corresponding service level is less than the one imposed (α=0.90). That's why, the minimum adopted in this situation is the one of the case (-3, +3) for which the total cost is 266552,14 *um* and the service level is equal to 98.33%. We note that, the total cost without transfer is 304998,87 *um*. These results show the important effect of the degradation evolution over the horizon *H*.

CONLUSION

In this paper, we dealt with an industrial problem of an enterprise facing a random demand on a finite horizon and given a certain service level. The manufacturing system cannot satisfy all the demand throughout the horizon. For the same reason it calls upon subcontracting. The key of this study is to consider that the failure rate increases with time and according to the production rate. Firstly, we formulated and solved a linear-quadratic stochastic production problem to obtain a production plan. Using the HMMS model, the plan minimizes the production and the inventory cost with a variable production rate. The plan also defines the production rate for manufacturing systems, contractor and subcontractor, during each period over the production horizon. In a next stage, we introduced a preventive maintenance strategy. Starting from the previously defined production plan, the study aims at optimizing simultaneously the same production plan and its newly introduced maintenance policy. The objectives are finding out the partition number of the production horizon *H* after which a preventive maintenance is required and, defining the transferable quantity from the contractor machine to its subcontractor counterpart. Through this study we proposed an analytical model to meet a random demand over a finite horizon incorporating subcontracting constraint. The study proposes an optimal production plan by minimizing simultaneously the production, the inventory and the maintenance costs. The model shows that by subcontracting part of the burden to produce, in addition to occurring less degradation in the manufacturing system, there are occasions where subcontracting is effectively more economically profitable then working with the maximum production rate. As a perspective to this study, we propose to progress with an imperfect and non negligible duration of preventive maintenance policy, then, assess its impact on the optimal maintenance/production plan.

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