

## A NUMERICAL OPTIMISATION OF A STIRLING ENGINE

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**Abstract:** *A zero dimensional finite-time thermodynamic, three zones model of a generic Stirling engine has been developed and experimentally validated. Taking into account time-dependant heat transfers and losses, the model calculates the evolution of gas temperature, mass and pressure in each zone. The architecture of the model, an assembly of blocks, permits an easy change of engine characteristics, geometries, gas proprieties. It is therefore possible to test rapidly various modifications on the numerical engine to see their impact on the engine performances and so manufacturing only it seems interesting.*

**Keywords:** Stirling engine, model, experimental validation, parametric study, numerical optimisation

### INTRODUCTION

In the present energy supply reflexion it is necessary to find alternative energy sources, for transports as well as for domestic uses. Two axes can be considered, the first concerns more efficient energy converters. The second, often less attended, concerns waste heat recovery, (1), from energy converters and use of low-quality energy renewable resources. An interesting energy converter for both objectives is the Stirling engine, (2,3), as it has the potential to provide high levels of efficiency and shows the potential to use energy sources at low temperature or with low-heating values. Its development has suffered for years from the extensive use of oil-burning in internal combustion engines as low-cost available energy.

Stirling engine can be adapted to different applications. In mobility applications, the hybridisation can get round its lack of flexibility (4). In this perspective the efficiency/weight balance of a thermal engine is the main challenge facing future developments, (2). In stationary applications the possibility of the use of exhaust heat of existent devices, the use of locally available non-refined fuels, (5), and the use of low temperature sources is of main interest.

Stirling engines must be studied in the perspective of such challenges, incorporating innovative technologies on materials and design. The aim of the joint research laboratory between ISAT and Danielson engineering, ID-MOTION, concerning the Stirling engine for the future project is to develop a demonstrator of an advanced Stirling engine able to face the challenges of effective energy conversion. The present work aims, thanks to a model complete enough to simulate the effects of the change of a given set of parameters on efficiency and weight, but simple enough to allow exhaustive optimization of a set of parameters, to present how a Stirling engine can be designed to respond to a specific application with specific constraints.

Many work has been carried on Stirling machines, but relatively few models have been developed able to simulate realistic cycles and performances for a large range of functioning parameters values. It is the first step of the work presented, how the model is featured and the validation of its results by comparison with a model experiment. Next is presented an optimization approach aimed at providing the best set of construction and functioning parameters for a given application.

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## MODEL

The first step of this study was to develop a numerical model describing performances of Stirling engine. To be usable, this model must be faithful to the reality, and this, on a wide range of use. Constraints of this model were its flexibility in both terms of geometry engine and operating point. Modelling efforts on Stirling engines have been carried out by a considerable number of researchers (6,7,8), and as a device its functioning seems to contain no unknowns. The main objective of the model in the frame of this study is to provide a tool able to carry predictive trends of engine performance.

The originality of this model, contrary to Abbas who uses isothermal exchanger (9) or Kongtragool who does not take into account the time dependency of exchanges (10), is the implementation of time-dependant heat and mass transfers on three different considered zones. The three zones correspond to volumes exchanging heat with a hot source, a cold source, and a regenerator.

The core of the model is constituted by three thermodynamics equations, ideal gases [1], energy conservation [2], and mass conservation [3]. These three equations form a differential system resolved by Adams predictor-corrector method.

$$P_i \cdot dV_i + V_i \cdot dP_i = m_i \cdot r \cdot dT_i + r \cdot T_i \cdot dm_i \quad [1]$$

$$cv_i \cdot T_i \cdot dm_i + cv_i \cdot dT_i \cdot m_i = -P_i \cdot dV_i + \sum h_j \cdot dm_j + \delta Q_i \quad [2]$$

$$\sum dm_i = dm_{\text{leak}} \quad [3]$$

Where  $P_i$  is the pressure in the zone "i",  $V_i$  its volume,  $m_i$  its mass,  $T_i$  its temperature and  $r$  is the specific gas constant,  $cv_i$  is the specific heat at constant volume,  $h_j$  the enthalpy of the gas through the interface "j",  $\delta Q_i$  is the heat transferred to the gas.

Functions governing as well engine characteristics as operating conditions allow solving the system and calculating pressure, From the calculation of pressures, the model calculates numerical work [4], power [5] and efficiency [6] of the engine. It is therefore possible to evaluate performances of a given set of parameters (exchange coefficients, kinematics, and dimensions).

$$W = \sum (P_c \cdot dV_c + P_h \cdot dV_h) \quad [4]$$

$$P = W \times \frac{N}{60} \quad [5]$$

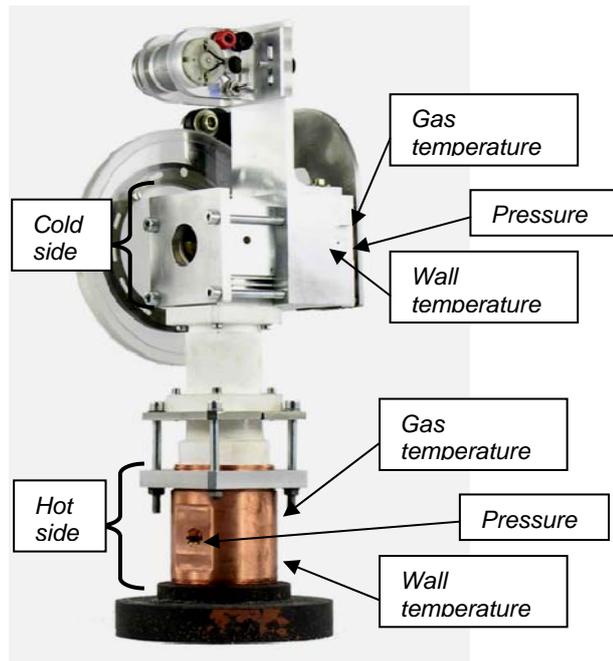
$$\mu = \frac{W}{Q_{\text{in}}} \quad [6]$$

Where  $c$  represents the cold side and  $h$  the hot one,  $N$  the rotational velocity and  $Q_{\text{in}}$  the energy brought to the engine.

## MODEL VALIDATION

A Stirling engine was instrumented to get experimental data allowing model validation. The experimental rig is a little gamma Stirling engine, see

Figure 1, with one gas pressure sensor, one gas temperature and one wall temperature sensor in each hot and cold volume. An electrical engine and a gas burner allow to control respectively the rotation speed and the hot temperature of the engine.

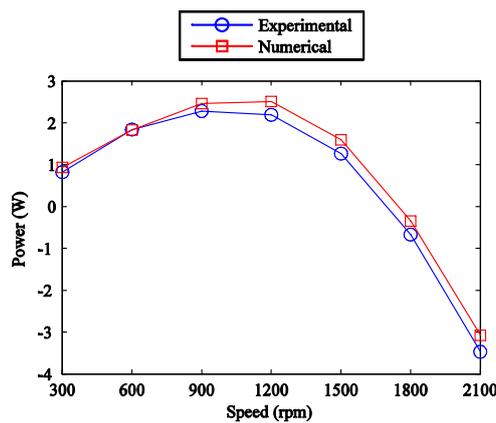


**Figure 1.** Engine for the validation

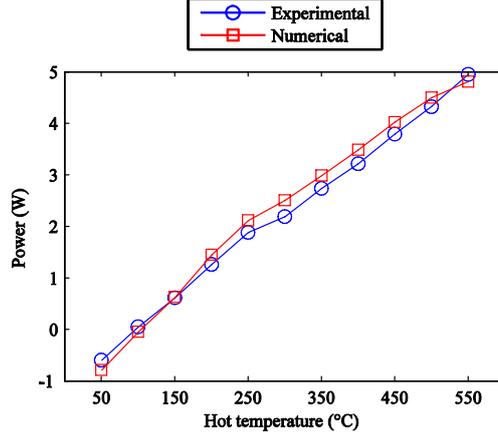
Exact dimensions of the engine were put on the model to calculate its numerical performance. Experimental conditions of each acquisition points were included in the model to get numerical power comparable to experimental power.

The engine was tested, and the model validated, for rotational speeds from 300 to 2100 rpm and hot wall temperatures from 50°C to 550°C, the cold one staying near 35°C.

On Figure 2 we can see that evolutions of the numerical and experimental power in function of the engine speed with a hot temperature of 300°C are very close. It is the same for the power evolution in function of the hot temperature, Figure 3, with an engine speed of 1200 rpm.



**Figure 2.** Power Evolution in Function of the Speed



**Figure 3.** Power Evolution in Function of the Hot Temperature

Figure 2 and Figure 3 represent only some test points but they reflect the model behaviour for a large range. This reveals that the model gives a good idea of the engine performance. This good agreement, added to its flexibility, enables to test a large variety of engine configurations. So the model we have developed is a real optimisation tool which allows trying various engine configurations to find the optimal one without making many parts and tests.

### OPTIMISATION ALGORITHM

The search of an optimal configuration generates thousands of calculations. An optimisation algorithm is used to launch calculation automatically. The algorithm we choose to use is the Particle Swarm Optimization (PSO), (11). It was invented by Russell Eberhart and James Kennedy in 1995, (12). Initially it was created to simulate social behaviour but it can be used to a multiple parameters optimisation.

The principle of this algorithm is to generate a population in a space of n dimensions, where n is the number of parameters. Each particle has a set of parameters which gives one result related to an efficiency criterion. In function of its result and those of the other particles, they move toward the optimum.

Each particle  $i$  has an initial position  $x_i$  and an initial velocity  $v_i$ . One time step later the new velocity of each particle is calculated as a function of the precedent positions and results associated, [7]. The new position is updated, [8].

$$v_i = v_i + rand * (xp_i - x_i) + rand * (xg - x_i) \quad [7]$$

$$x_i = x_i + v_i \quad [8]$$

$xp_i$  is the optimal position of i-th particle and  $xg$  the global optimal position. Differences between the actual position of each particle and its optimal, and the global optimal, with a random coefficient, allows defining a new velocity and so a new position. Figure 4 presents the algorithm sequence.

In our case each particle represents one engine configuration and the result we want to maximize is the efficiency (it could also be power, torque, volume, weight, etc.). In this study we search to optimize some dimensions of the engine, see Figure 5, that re the power piston bore, the two pistons stroke, the two pistons rod and the dead volume of the power cylinder.

The optimisation we have realised starts with an initial engine. To have a representative study it is necessary to fix one parameter in all configurations, permitting, that way, to do the comparison. We choose to keep the sweeping volume constant, [9].

$$\frac{\pi}{4} \times piston\ bore^2 \times piston\ stroke \quad [9]$$

For each parameter we define maximal length, the minimum being zero for each length. In this interval, the optimisation algorithm defines, with a random part, the history of the particle and of the swarm, a set of parameters to strive for the optimal setting.

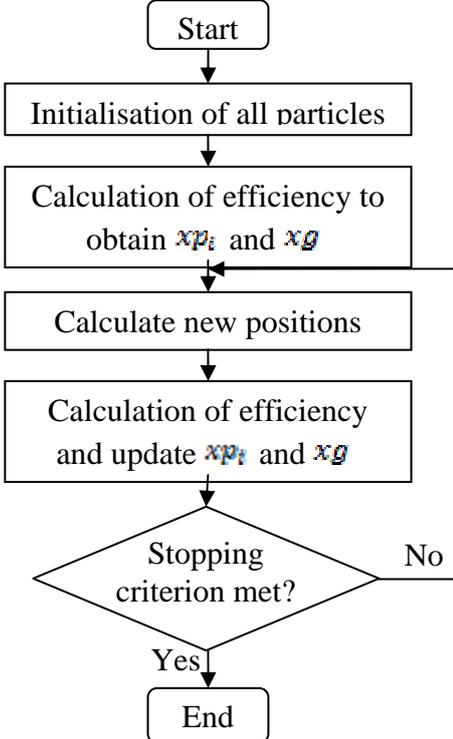


Figure 4. PSO flowchart

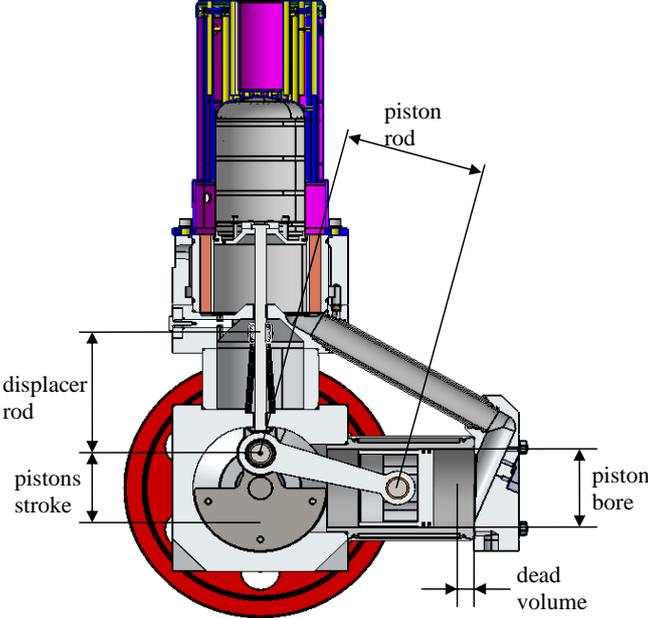


Figure 5. Engine parameters

**RESULTS AND DISCUSSION**

The engine has been numerically optimised for two operating points. For both the mean pressure is at 1 bar, the rotational speed at 800 rpm and the cold temperature at 35°C. The hot temperature of the first point is at 300°C and the one of the second is at 550°C.

**Table 1. Optimisation to 300°C**

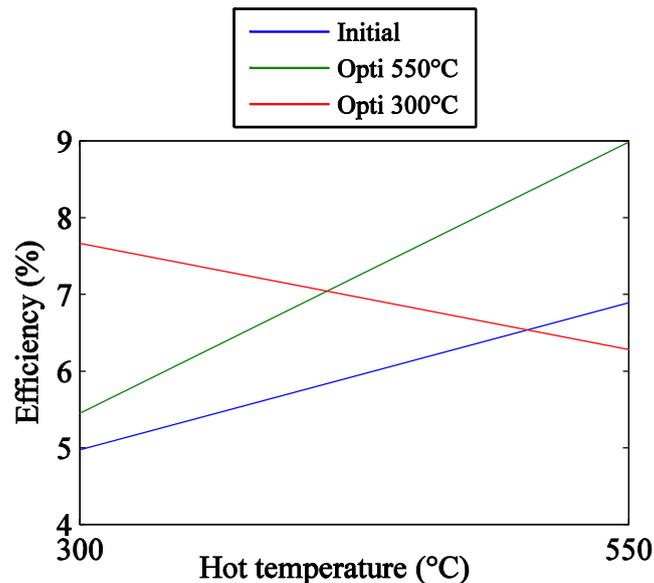
	stroke (mm)	bore (mm)	piston rod (mm)	displacer rod (mm)	dead volume (mm)	efficiency (%)
Initial	75	85	146	130	5.1	4,97
Optimised	47,84	106,42	119,36	120,82	0	7,66
Gap (%)	-36.21	25.20	-18.25	-7.06	-100	54.12

**Table 2. Optimisation to 550°C**

	stroke (mm)	bore (mm)	piston rod (mm)	displacer rod (mm)	dead volume (mm)	efficiency (%)
Initial	75	85	146	130	5.1	6,89
Optimised	72,59	86,40	148,73	165,09	0	8,98
Gap (%)	-3.21	1.65	1.87	26.99	-100	30.33

On Table 1 and Table 2 are noted initial engine configurations and the optimised ones, respectively at 300°C and 550°C, and their respective efficiency. If for both the stroke decreases and the bore increases, this tendency is more important as the temperature decreases. It is different for the length of the rods, at 300°C both rods are smaller than for the initial configuration whereas, at 550°C, they are longer.

The Figure 6 shows the variation of the efficiency with the two hot temperatures for each three previous configurations. We can see that the optimal configuration to one operating point is not the optimal to one another. If we want to design an engine to a precise point of functioning it is sufficient, but if we want a more flexible engine we must find a compromise.

**Figure 6.** Efficiency of each configuration in function of the hot temperature

## CONCLUSION

In this article we present a numerical one dimensional, three zones model of Stirling engine which was validated experimentally. The model shows a good concordance with the experimental trends over the entire range of parameters tested. It covers a range of hot temperatures from 50 to 550°C, rotational speeds of 300 to 2100 rpm. It brings out that the model can predict correctly the influence of operating parameters.

The second part of this study present how this model can be very useful in the design of Stirling engines. In a first step it is possible to simulate many conditions, temperature, speed, and find the optimal rate. In a second step, for fixed conditions, the model can help to determine optimal engine dimensions and kinematics.

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