

## DETERMINATION OF VIBRATION TRANSMISSIBILITY OF THE SUSPENSION MECHANISM IN THE STRUCTURE OF CAR AREA STABILITY

Helene SUSTER BADARAU\*  
University of Pitesti, Romania

**Abstract:** *In this paper analyzes the stability and convenience vehicle, determined by how its suspension, wheel and finishing from the driver's seat, made his leadership ability in all road conditions or weather-climate.*

*Thus the vehicle suspension, as the linking ground and its body body, so that taking over and share all the interaction forces and movements of the wheel with the road, has a role in acquisition, transmission and filtering them, which in ultimately turns into creating awareness of driving comfort. Study using the model of the car with two degrees of freedom and analytical method for assessing the quality of vehicle suspension, frequency response and root mean square response system were obtained indicators for assessing the sensitivity factors for excitation system external and mode of transmission.*

*Also, the analysis of this model study, we obtained information on the area of stability, determining the alteration of the mass distribution coefficient suspended, once determined its position on the surface of the car, suspension parameters were analyzed and their interdependence response to changes in function of the system to internal and external disturbance, analyzing the response curves represent three-dimensional functions, as seen in this link is better and achieving maximum efficiency in the optimization of the characteristics of driving.*

**Keywords:** Driving capacity at automotive, driving, road, suspension transmissibility, stiffness and rigidity.

### DETERMINATION OF VIBRATION TRANSMISSIBILITY OF THE SUSPENSION MECHANISM IN THE STRUCTURE OF CAR AREA STABILITY

Stability and convenience of a car is determined by how its suspension, wheel and finishing from the driver's seat, made his leadership ability in all road conditions or weather-climate. From this point of view the car suspension, as the linking ground and his body body, so that taking over and share all the interaction forces and movements of the wheel with the road, has a role in the acquisition, transmission and their filtering, which ultimately turns into creating awareness of driving comfort.

To create a maximum degree of comfort required that the suspension is very supple, so have a very low stiffness with a damping coefficient of suspension forces and vibration of very small (slow depreciation over time), which is in contradiction with requirements concerning the issue of wheel-ground grip, stability and maneuverability of the car when driving on rough terrain, where: to have a good stability requires that depreciation be strong to take quick and effective forces and shocks that occur in suspension when driving off-road vehicle in such conditions (coefficients of stiffness  $k$  and damping  $c$  of the suspension, to be large).

Addressing this goal is usually done through a compromise between comfort and stability, maneuverability, and the modern cars by adopting so-called active suspension or interactive, but requires a complicated construction and electronic control of movement parameters (Electronic Control system suspension-CES, able to take into account the stiffness of the suspension, along with position and race to the car body). So to meet the above requirements will first have to know how to takeover,

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\* Corresponding author. Email: helene.suster@upit.ro

transmission and damping of movements and vibrations introduced into the vehicle structure from wheel-road interaction.

In this analysis, the coefficient of rigidity of spring damping coefficient  $k_s$  and  $c_s$  are considered as variable suspension obtained in response to excitation and analysis of indicators analyzed value is independent of these factors. Vibration analysis due to the interaction elements wheel-road suspension, will be the same race suspension for the data to be comparable.

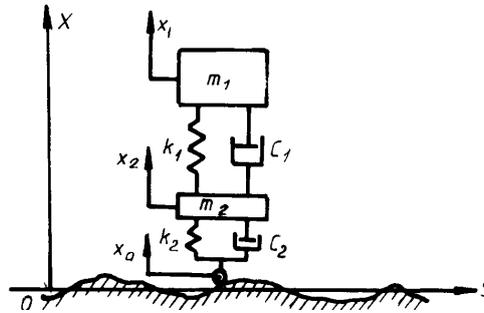


Figure 1. Car model

Study using the model of the car with two degrees of freedom in Figure 1 and analytical method for assessing the quality of vehicle suspension, frequency response and root mean square response (transmissibility) system, to obtain indicators for assessing the sensitivity factors for excitation system and mode of transmission of the phenomenon analyzed.

Also, the analysis of this type of study will obtain information on the area of stability (the area where the center of gravity moves the car during its oscillation, caused by alteration of the suspended mass distribution coefficient ) determined its position vehicle surface, specifically considering the suspension parameters and their interdependence to change the system response function to internal and external disturbance. To have a more complete and relevant factors influencing the characteristics of interdependence suspension, will present curves representing three-dimensional response functions, thus observing the connection is better and achieving maximum efficiency in the Optimization of suspension characteristics.

### ANALYSIS OF FACTORS INFLUENCING THE CHARACTERISTICS OF THE SUSPENSION, USING THE MODEL WITH TWO DEGREES OF FREEDOM STUDY

The study with two degrees of freedom of the car, is a Linear model, the coefficient of rigidity of spring  $k_1$  and  $c_1$  of the suspension damping coefficient are linear, and for our tire stiffness coefficient  $k_0$  and  $c_0$  damping coefficient is very small compared with  $c_1$  zero, this simplification leads to a clearer picture of the quality of the suspension.

Writing the equations of equilibrium for the masses of the two bridges, we obtain:

$$\begin{aligned} m_1 \ddot{Z}_1 + c_1(\dot{Z}_1 - \dot{Z}_5) + k_1(Z_1 - Z_5) &= 0 \\ m_2 \ddot{Z}_5 - c_1(\dot{Z}_1 - \dot{Z}_5) - k_1(Z_1 - Z_5) + k_2(Z_5 - Z_{01}) &= 0 \end{aligned} \quad (1)$$

with the variables:

- $x_1 = Z_1 - Z_5$  -strain (deformation) suspension;
- $x_2 = \dot{Z}_1$  -speed mass suspended;
- $x_3 = Z_5 - Z_{01}$  -strain (deformation) tires;
- $x_4 = \dot{Z}_5$  -speed of mass unsprung;

Equation (1) can be rewritten taking into account the defining (2);

$$X = AX + LZ_{01} \quad (2)$$

where:

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}; A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -k_1/m_1 & -c_1/m_1 & 0 & c_1/m_1 \\ 0 & 0 & 0 & 1 \\ k_1/m_2 & c_1/m_2 & -k_2/m_2 & -c_2/m_2 \end{bmatrix}; L = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad (3)$$

Index to evaluate transmissibility of this system is given by:

$$J = \lim_{T \rightarrow \infty} \frac{1}{T} E \left[ \int_0^T \left\{ \ddot{Z}_1^2 + \rho_1 (Z_1 - Z_5)^2 + \rho_2 \dot{Z}_1^2 + \rho_3 (Z_5 - Z_{01})^2 + \rho_4 \dot{Z}_5^2 \right\} dt \right] \quad (4)$$

Where:  $\rho_1, \rho_2, \rho_3, \rho_4$  -weight-factor; T -vibration period; t -function of time; and equation (4) can be rewritten as;

$$J = \lim_{T \rightarrow \infty} \frac{1}{T} E \left[ \int_0^T (X^T Q X) dt \right] \quad (5)$$

where:

$$Q = \begin{bmatrix} k_1^2/m_1^2 + \rho_1 & c_1 \cdot k_1/m_1^2 & 0 & -c_1^2/m_1^2 \\ c_1^2/m_1^2 & c_1^2/m_1^2 + \rho_2 & 0 & c_1^2/m_1^2 \\ 0 & 0 & \rho_3 & 0 \\ c_1 \cdot k_1/m_1^2 & c_1^2/m_1^2 & 0 & c_1^2/m_1^2 + \rho_4 \end{bmatrix} \quad (6)$$

Admitting that the spectral density of random process is approximately constant over a wide frequency band, the so-called *white noise* and the system is linear hypothesis of the ergodicity be satisfied, then we can say that, while the average value is equal to the average overall, which in fact is often encountered in practice, justifying the hypothesis adopted.

A sufficient condition for a stationary random process  $x(t)$  be ergodic is that its covariance function  $C_x(\tau)$ , satisfy the following integrability properties:

$$\int_{-\infty}^{\infty} |C_x(\tau)| d\tau \leq \infty; \int_{-\infty}^{\infty} C_x^2(\tau) d\tau \leq \infty; \int_{-\infty}^{\infty} |\tau \cdot C_x(\tau)| d\tau \leq \infty; \int_{-\infty}^{\infty} |\tau| \cdot C_x^2(\tau) d\tau \leq \infty; \quad (7)$$

These four relations are satisfied:

$$J = \lim_{T \rightarrow \infty} \frac{1}{T} E \left[ \int_0^T (X^T Q X) dt \right] = \lim_{T \rightarrow \infty} \frac{1}{T} \left[ \int_0^T (E[X^T Q X]) dt \right] = [QE[X^T X]] = [Q\Sigma] \quad (8)$$

where:  $\Sigma$  is the covariance matrix of the form;

$$\Sigma = E[XX^T] = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} & \sigma_{34} \\ \sigma_{14} & \sigma_{24} & \sigma_{34} & \sigma_{44} \end{bmatrix} \quad (9)$$

Covariance function equation (3), we can write the following form:

$$A \Sigma + \Sigma A^T + L \Sigma L^T = 0 \quad (10)$$

Covariance function equation (10), gives the mean square acceleration of unsprung mass, the deflection of suspension and deflection of tyre:

$$\bar{x}_2 = \sqrt{\text{Var}[\dot{x}_2]} = \frac{1}{m_1} \left( E[k_1^2 x_1^2 + c_1^2 x_2^2 + c_1^2 x_4^2 + 2c_1 k_1 x_1 x_2 - 2c_1 k_1 x_1 x_4 - 2c_1^2 x_2 x_4] \right)^{1/2} = \frac{1}{m_1} \left( k_1^2 \sigma_{11} + c_1^2 \sigma_{22} + c_1^2 \sigma_{44} + 2c_1 k_1 \sigma_{11} - 2c_1 k_1 \sigma_{14} - 2c_1^2 \sigma_{24} \right)^{1/2}; \quad (11)$$

$$\bar{x}_1 = \sqrt{\text{Var}[x_1]} = \sqrt{\sigma_{11}}; \bar{x}_3 = \sqrt{\text{Var}[x_3]} = \sqrt{\sigma_{33}};$$

Function J is calculated using the covariance matrix and we get:

$$J = 1/m_1^2 (k_1^2 \sigma_{11} + c_1^2 \sigma_{22} + c_1^2 \sigma_{44} + 2c_1 k_1 \sigma_{12} - c_1 k_1 \sigma_{14} - 2c_1^2 \sigma_{24}) + \rho_1 \sigma_{11} + \rho_2 \sigma_{22} + \rho_3 \sigma_{33} + \rho_4 \sigma_{44}; \quad (12)$$

### SENSITIVITY ANALYSIS-EVALUATION SYSTEM RESPONSE FREQUENCY RESPONSE FUNCTION

The first analysis is made, modifying the characteristics of suspension parameters and analyzing the frequency response of the system model considered, at different speeds external excitation system input.

Changing the spring stiffness coefficient  $k_s$  and the suspension damping coefficient  $c_s$  will be for five cases: 25, 50, 100, 200 and 400% of the nominal value of the parameter analyzed. In figures 2, 3 and 4, shows how the variation of frequency response, the suspended mass, the suspension and tires, changing the stiffness and damping coefficient of suspension. Thus in figure 2 shows how the variation of the frequency response of the sprung mass acceleration, ranging  $k_s$  (coefficient of rigidity of the spring suspension) and  $c_s$  (the suspension damping coefficient), direction of the arrow showing the direction of increasing parameter changed.

The analysis diagram can be observed that the increase in damping reduces the response to the first part of the chart, but increases at high frequencies (see fig.2.a) and from (fig.2.b) we see that the effect of stiffness variation suspension is not important values in the natural frequencies (base) of the suspension, but is highest around the values of first and second mode of vibration.

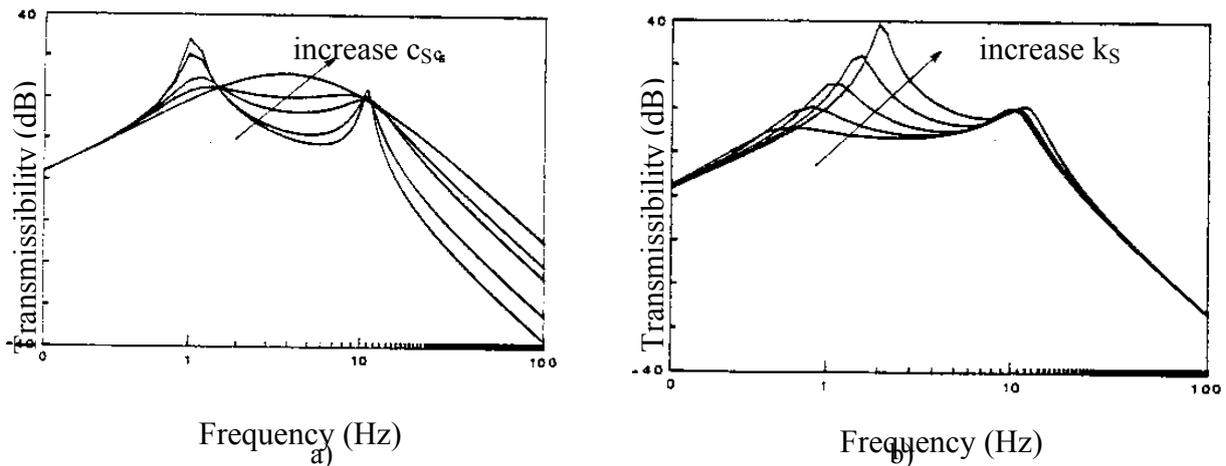


Figure 2. Transmissibility vs. frequency

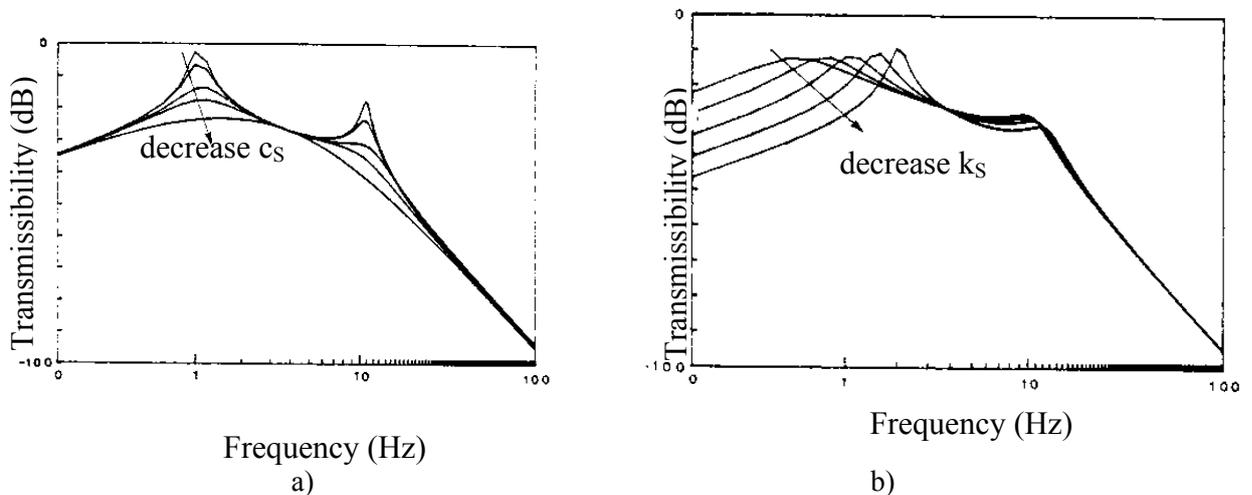


Figure 3. Transmissibility vs. frequency

The influence of suspension stiffness changes on transmissibility diagram shown in figure 3, where it notes that, to increase damping decreases the transmissibility (Fig.a) and the (fig.b) observed that with increasing stiffness suspension strong amplification occurs and also a shift of its maximum under the first mode of vibration in the 4-10 Hz and 2-3 Hz in the high frequency area, there is mitigation.

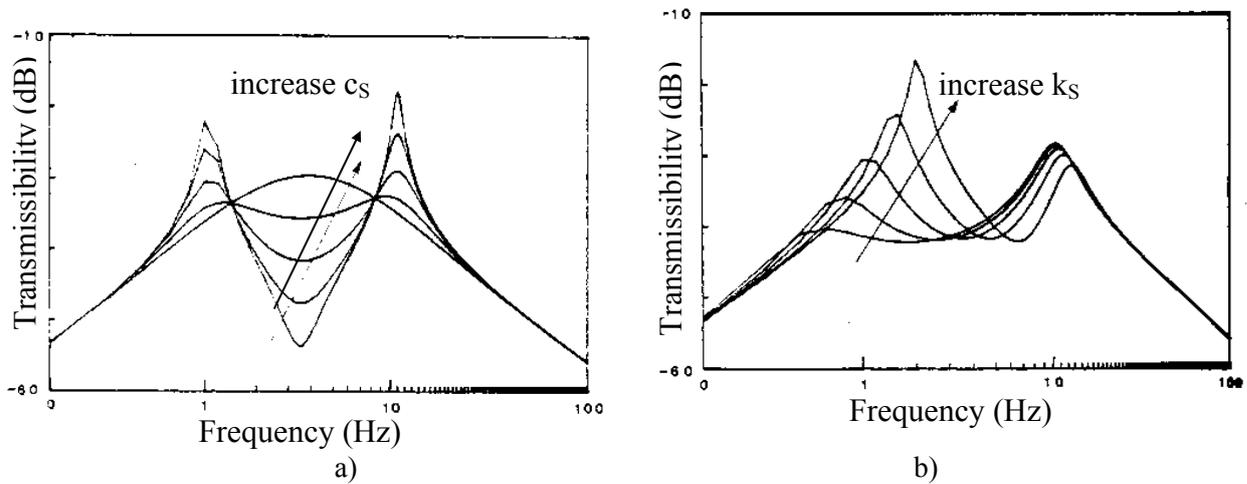


Figure 4. Transmissibility vs. frequency

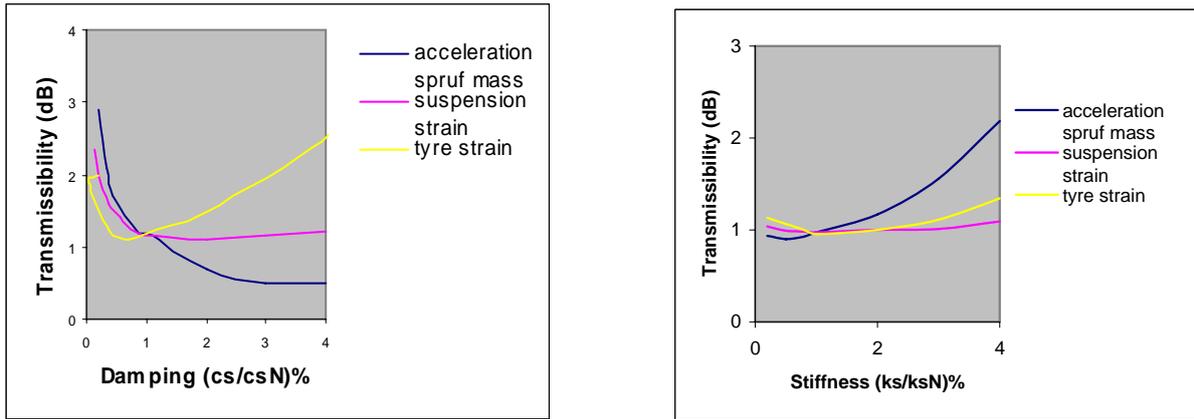
The influence of tire stiffness changes on transmissibility is shown in figure 4 and observe that the damping coefficient lower level  $c_s$  is increasing response function in both modes of vibration, the effects of changing between the two modes of vibration.

Thus with increasing damping, the model approaches the model with a single table with a single vibrational mode at about 4 Hz. Change of  $k_s$  (coefficient of rigidity of the suspension), has a strong influence on the transmissibility so that the spring stiffness increase occurs immediately increased transmissibility and also the working frequency of the suspension, and to decrease its attenuation occurs in the low frequencies with a slight change in the high frequencies.

### EVALUATION OF TRANSMISSIBILITY

For the analysis results are given several cases, noting the parameters analyzed at different frequencies, which is an important factor in the analysis of transmissibility. For this reason transmissibility (response function were the root mean square response system) is calculated for each parameter measured. This response is shown in the diagram of figure 5, depending on the parameters of the suspension. All diagrams transmissibility (response function) are normalized values for achieving nominal position of the suspension.

Diagram transmissibility function (root mean square response), taking into account three parameters and vary the suspension damping coefficient  $c_s$  is shown in figure 5 a, marking the nominal point N. This point is a good compromise between isolation and road. It is impossible to improve performance by reducing the suspension, sprung mass acceleration, suspension or tire stiffness, without modifying one or other of the parameters or both. Influence on the parameters of the suspension, modifying the stiffness of the spring coefficient  $k_s$  is presented in figure 5.b, where it notes that the transmissibility function is dependent  $k_s$  strain suspension (suspension stiffness) and again by examining figure 3 b, we see that the frequency response varies with the stiffness of suspension spring stiffness of the suspension. This influence is visible attenuation-amplification over all frequencies to zero in the middle so that the root mean square value remains constant.



a) b)  
Figure 5. Transmissibility vs. damping and stiffness

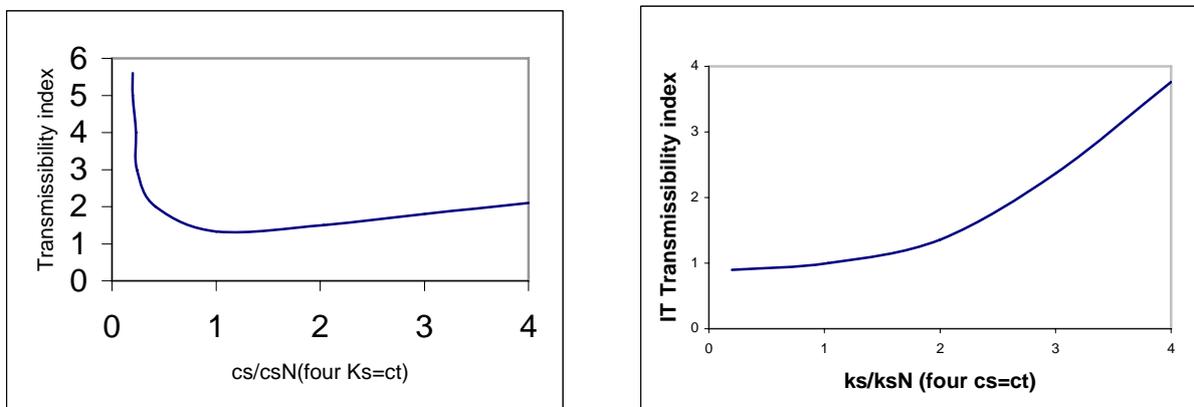
Also in figure 5.b), shows increasing isolation (factor reducing transmissibility of the sprung mass acceleration), that can be achieved by reduction of  $k_s$ , without visibly affecting the tire stiffness. Although some are effective for soft springs, then you have to take into account external forces behavior, body size or weight of its body. Although some are effective for soft springs, then you must consider the behavior to external forces, body weight and size of its body.

The next step in this analysis is done by changing two parameters simultaneously, rising three-dimensional graphics in the system and examined the surface thus created variation in response mode, varying one of the parameters directly. This chart has the advantage that allows easy interpretation and use of parameters to optimize their suspension.

**EVALUATION INDICATORS TRANSMISSIBILITY**

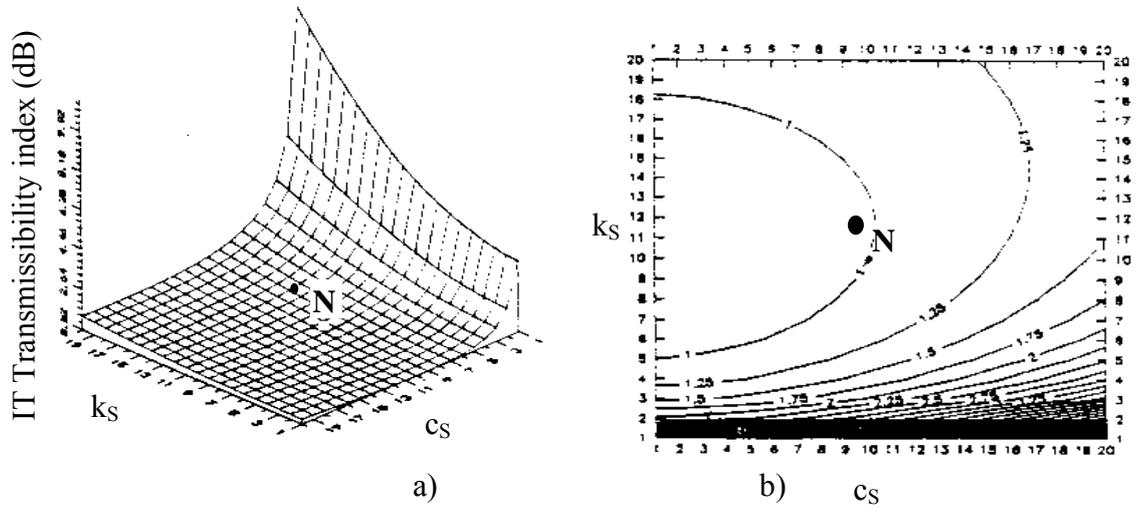
Indicator for evaluating IT transmissibility, which is defined by equation (4), which take into account two variables of the suspension stiffness coefficient of the damping coefficient  $k_s$  and  $c_s$ , the results are presented graphically in figure 6 a and b. The analysis curve  $c_s$  damping coefficient variation is a parabola convex type for that  $k_s = ct$ , the optimal damping coefficient  $c_s$  is when IT is minimal, resulting in maximum performance.

In figure 6.b is shown a curve increasing monotonically with increasing  $k_s$  where: for low values of stiffness spring suspension to obtain high levels of transmissibility index.



a. b.

Figure 6. Transmissibility index vs. damping and stiffness



**Figure 7.** Three-dimensional coordinates obtained by changing simultaneously the surface coefficient of spring stiffness and damping coefficient  $k_s$ ;  $c_s$ .

In figure 7 is presented in three-dimensional coordinates obtained by changing simultaneously the surface coefficient of spring stiffness and damping coefficient  $k_s$ ;  $c_s$ .

The surface is divided into 20 steps range from 10-200% of nominal value, point N is the nominal value.

From this figure, the resulting parameter with the greatest influence on the transmissibility of the suspension index, this is the spring stiffness and is compared with the curves of figure 6, we see that the performances are strongly dominated by the spring stiffness coefficient.

Diagram allows to obtain the spring stiffness coefficient when depreciation is optimal, resulting in maximum performance suspension. These results are useful for optimization of suspension parameters in the design process.

## SIMULATION OF COMFORT AREA, USING THE COMPLETE MODEL OF THE CAR STUDY

### Formulating and determining the area of comfort

Use complete dynamic model of the car study with 7 degrees of freedom are, 3 for data table suspended center of gravity (vertical motion, pitch and roll) and 4 for unsprung masses in the centers of gravity of the bridges to and back (vertical and rocking motion, rigid axle and two vertical, if independent suspension).

Where to write equations suspended and unsprung mass displacements as:

$$Z_1 = \begin{Bmatrix} Z \\ \theta \\ \Phi \end{Bmatrix} \quad Z_5 = \begin{Bmatrix} Z_5 \\ Z_6 \\ Z_{7,8} \\ \beta'' \end{Bmatrix} \quad (13)$$

Using vector calculation model of the system matrix and analyzed results:

$$\begin{bmatrix} M & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{Z} \\ \ddot{z} \end{Bmatrix} + \begin{bmatrix} RCR^T & -RC \\ -CR^T & C \end{bmatrix} \begin{Bmatrix} \dot{Z} \\ \dot{z} \end{Bmatrix} + \begin{bmatrix} RKR^T & RK \\ -KR^T & K+k \end{bmatrix} \begin{Bmatrix} Z \\ z \end{Bmatrix} = \begin{Bmatrix} 0 \\ kz \end{Bmatrix}; \quad (14)$$

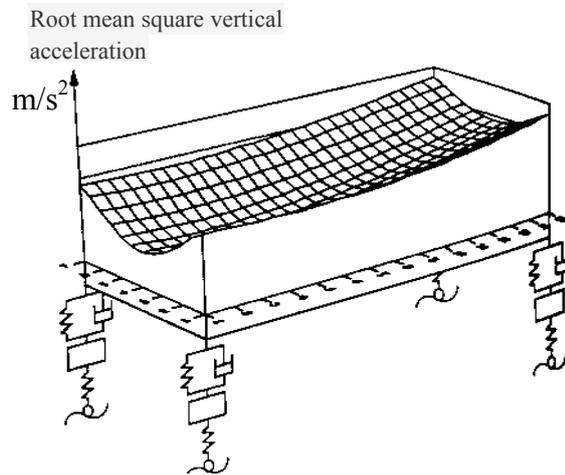
where:  $M$  is the mass of suspended,  $-Z$  is vertical movement of the suspended mass,  $m$  -is unsprung mass,  $z$  -is the vertical movement of the unsprung mass,  $\ddot{Z}$  -is mass vertical acceleration, is suspended vertically,  $\ddot{z}$  -is the acceleration of unsprung mass.

After the modal analysis of these equations, transient response is calculated for the so-called stationary *white noise* processes, the model taking into account the road profile. Graphic network is divided into 105 points (15x7), consisting of 15 divisions in length and 7 width of the car. Values of acceleration transmissibility function vertical movements: lift, roll and pitch are shown in figure 8. Transmissibility function curves are convex parabolic shape in the longitudinal and transverse, most hovering around the center of gravity near the midline where the transmissibility function has the highest values.

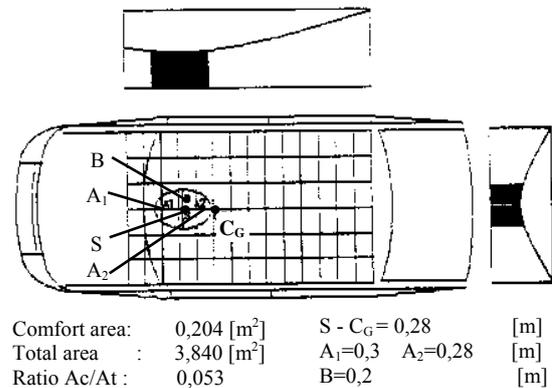
Also, there must be a minimum of transmissibility function values and can be found where the lowest level in the region surrounding the point format (S).

*Area of stability*, or as he says, the *area of comfort* is defined as follows:

Area of stability for the basic model is determined by rates  $A_1$ ,  $A_2$  and  $B$  which determine the distribution function of transmissibility level (see fig. 9), this finding is the point  $S$  where: is the minimum level of acceleration on the center line and determine  $A_1$ , which is more than 50% of the focus area calculated by integrating the distribution curve point  $S$  and  $B$  similar cross.



**Figure 8.** Values transmissibility function of vertical acceleration



**Figure 9.** Distribution function of transmissibility level

Follow the determination of  $A_2$ , calculated as  $A_1$  and  $B$  and up to 20% of the total integrated behind surfaces axis  $S$ . The distribution point is uniformly distributed comfortable area where  $A_1$  and  $A_2$ , representing an ellipse around the point  $S$ , the calculations being made for the nominal position to have for comparison.

### Assessment of area of stability (comfort), the system response

In this study tend to take account of changing the area of stability, defined in advance by varying system response function analysis, the coefficient of rigidity of spring damping coefficient  $k_s$  and  $c_s$  analyzed the response function, calculated in 20 steps for a variation of 10-200% of nominal value.

### CONCLUSIONS

Arch suspension and damping coefficient are selected as response variables and function analysis of the results shows that:

1-Frequency and transmissibility are measured by analyzing the response function and are analyzed in isolation (as road) and moving the highly uneven road.

2-Change is viewed three-dimensional suspension parameters to estimate the optimal value and hence the conclusion need active suspension.

3-to examine the suspension, considering the area of comfort, knowing the dependence of the damping coefficient and the coefficient of rigidity  $c_s$   $k_s$  arc and also decreasing and increasing dependence of its damping coefficient of variation.

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