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THE RUNNING SIMULATION OF A HYBRID CAR

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ABSTRACT – The paper presents an experimental stand that simulates the running of a hybrid car. The important piece of the stand is a mechanical system which includes a planetary mechanism. It engages a heat engine, a three-phase asynchronous electric motor and a power alternator. The stand was designed to be mounted on a roller stand and a stand with electric brake. The mechanical system design was made in the CAD soft, which has facilitated the rapid achievement of the geometrical and mechanical measures necessary to simulate the mechanism functioning with two degrees of mobility that engage the three sources of power. The simulation was done with a computer program that allows solving the machine differential equations of motion. Previously there were presented the mechanism equations of motion with two degrees of mobility that engage the three sources of power. In the end are presented the conclusions derived from the numerical and experimental measurements.

THE KINEMATIC DIAGRAM OF THE COUPLING MECHANICAL SYSTEM

The hybrid self-propelling cars is done using a heat engine (MT) and an electric motor (EM) which is powered by a battery of high voltage batteries. The battery is charging even when the car is moving using an electric generator (GE) driven by the heat engine. The power sources (the heat engine, the electric generator, the electric engine) can be engaged using a planetary gear with two degrees of freedom, like in figure 1.

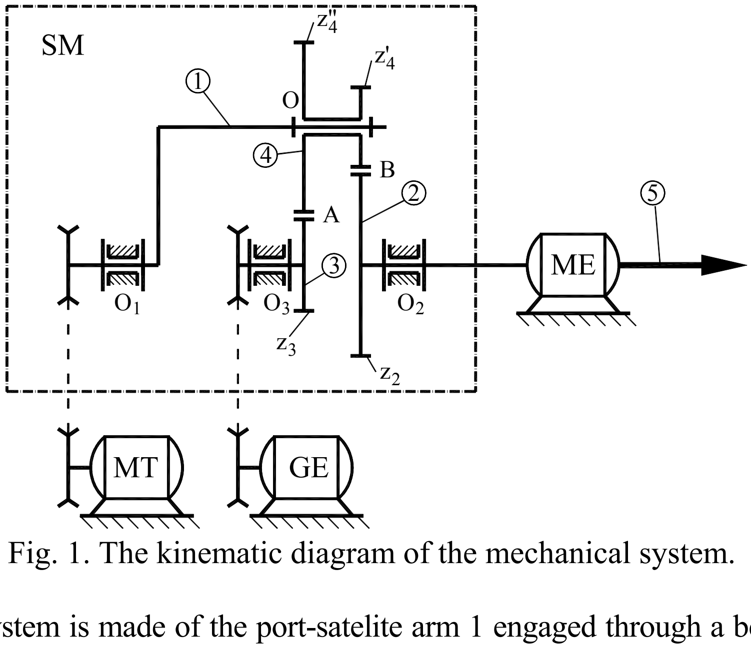

Fig. 1. The kinematic diagram of the mechanical system.

The mechanical system is made of the port-satelite arm 1 engaged through a belt transmission to the MT heat engine, the solar gear engaged through a belt transmission to the ME electric engine, the solar gear 3 engaged through a belt transmission to the GE electric engine, to the double satellite pinions 4 and to the exit shaft 5.

From kinematical point of view, being known the number of teeth of gears 2, 3 and 4 which are noted with z_2, z_3 and respectively z'_4, z''_4, and with *i* the expression:

$$i = \frac{z_2 z''_4}{z_3 z'_4}, \tag{1}$$

it is obtained the connection relation between the angular speeds of the mechanism:

$$\omega_3 = \omega_1(1 - i) + \omega_2 i. \tag{2}$$

Taking into account the values of the three engines' power, for the transmission ration *i* was chosen value $i = 2{,}6$.

Considering relation 1 and choosing for $z_3 = 20$ teeth, it will result: $z_2 = 35$ teeth, $z'_4 = 30$ teeth and $z''_4 = 45$ teeth.

Replacing in relation 1, in the end it results:

$$i = \frac{z_2 z''_4}{z_3 z'_4} = \frac{35 \cdot 45}{20 \cdot 30} = 2{,}625. \tag{3}$$

THE CONSTRUCTIVE DESIGN OF THE STAND

From the constructive point of view, the stand consists of a metallic frame on which is put a metallic plate with the elements of the mechanical system (fig. 2).

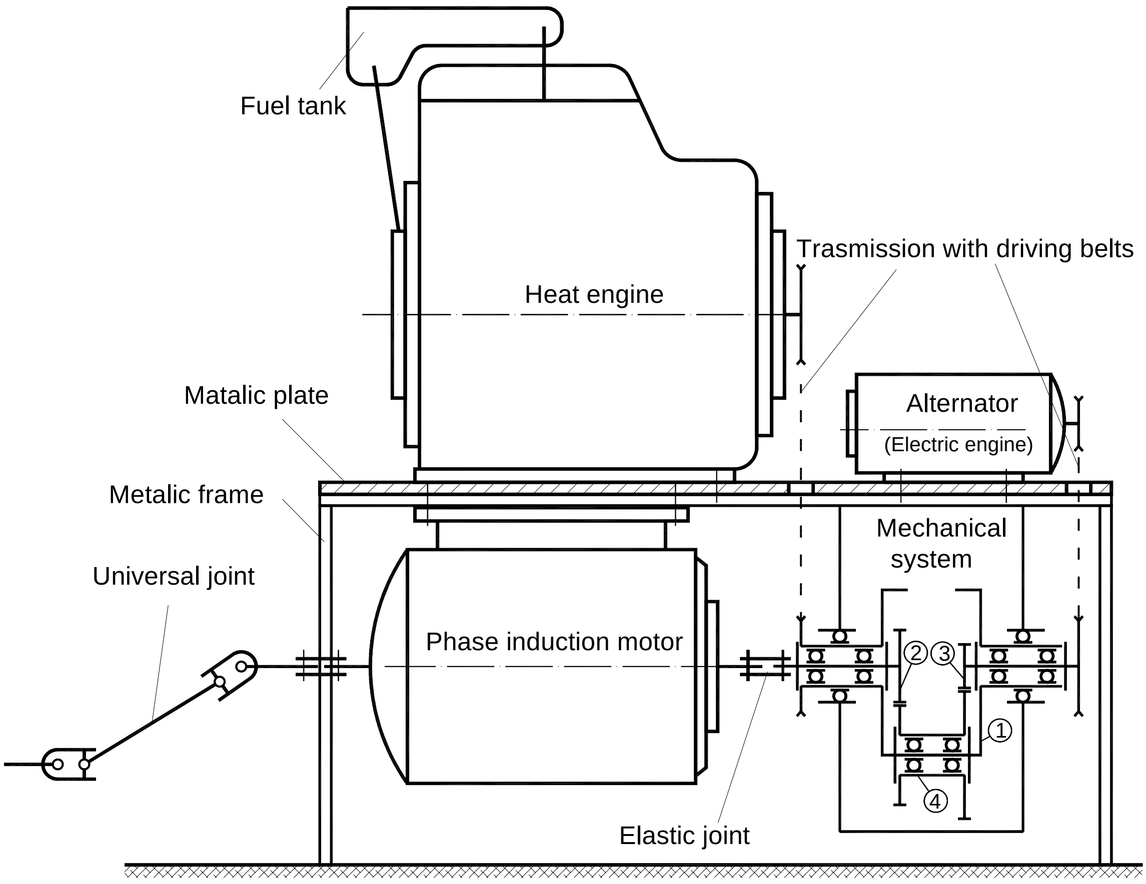

Fig. 2. The constructive design of the stand.

On the metallic plate are mounted by fastening with screws the elements of the stand: the mechanical coupling system of the power sources, an air-cooled heat engine fully equipped, a three-phased both sides exit electric motor and two air-cooled power alternators. The electric engine (MT from fig. 1) is engaged to the mechanical system through a belt transmission, being realised in this way the separation of the heat engine, from the vibrations point of view, from the rest of the mechanical system. From the mechanical point of view, the shaft of the heat engine is solitary to the rotation with the port-satelite arm of the planetary gear. The electric generator (GE din fig. 1) is a high output alternator or two medium-power alternators driven by belts jointly by the shaft solitary with gear 2. The generator is mounted on top of the assembly, on the same side as the heat engine. As shown in fig. 2, the electric engine ME is a three-phased engine with an output shaft on both sides. Such an approach leads to a compactness of the whole assembly, a good vibration isolation produced by the whole body in respect with the body of the car and, finally, to a small design of the way to locate the driving mechanism.

The heat engine is a gasoline engine HONDA type GX 670, with two cylinders in V on 90°, air cooling system, maximum power: 15,3 kW at 3600 rpm, maximum torque: 46,0 Nm at 2500 rpm, the electronic ignition system and electric starting system starter.

The electric engine is a three-phased electric engine with Type 132 S 38-2 with exit on both sides, 7.5 kW power and speed of synchronism 3000 rot/min at a frequency of 50Hz. The control module of the three-phased electric engine is composed of the power inverter SINAMICS and the command module 240S. SINAMICS power module allows asynchronous electric engine speed control by changing the supply voltage and using the frequency variation. For a correct functioning there are introduced from the start the voltage data, the maximum speed and the electrical engine power in the command module CU240S. Speed can be adjusted via a potentiometer and through a computer by scheduling the control mode operation.

The two power alternators are engaged through a belt to the shaft of the coupling mechanism, being mounted in the upper part of the assembly, on the same part with the heat engine. The alternators are of type A 002TCO982, Mitsubishi Electric with a nominal tension of 12V, a rectified tension of 14,5 V – 15,1 V and a maximum current of 210 A. The voltage of the two alternators is regulated by two voltage relays, each alternator discharging on its own battery. Next, two batteries battery voltage go either to consumers or to an inverter that transforms how continuous voltage three-phase AC. Next, the two batteries of accumulators are meant either for the consumers either for an inverter module that transforms the continuous tension into a three-phase alternative tension. It wasn't use such a module, the three-phase tension being available in the electrical network of the test lab.

In the current configuration, in which the generator is an alternator, it is necessary the electric start with a starter of the heat engine, the alternator not being a reversible electric car. Into the circuit of one of the batteries was introduced the control system of the heat engine electric starter. The start contact is also used for coupling the tension regulating relays. It was avoided the binding of the two batteries on the board, in order to separate the galvanic low voltage circuit of the one of power (380 V three-phased). Connections were made with two types of stranded copper conductors with diameters of 3.25 mm and 8 mm, depending on the nominal current of the circuit.

THE MECHANISM MOVEMENT EQUATIONS

The following notations are used:
$\theta_1, \theta_2, \theta_3$ the rotation angles of the shafts 1, 2, 3;

$\omega_1, \omega_2, \omega_3, \omega_4$ the absolute angular speeds of the elements 1, 2, 3;
J_1, J_2, J_3, J_4 the inertia moments of the elements 1, 2, 3;
i_1, i_2 the transmission ratios:

$$i_1 = \frac{z_2}{z_1^{'}}; i = \frac{z_2 z_4^{''}}{z_3 z_4^{'}} \quad (4)$$

M_1, M_2, M_3 the moments that act elements 1, 2, 3;
m_4 the 4 element mass;
R the crank *1* radius;
A, B, C the parameters defined by relations:

$$A = J_1 + m_4 R^2 + \left(1 + i_1^2\right) J_4 + (1 - i) J_3$$
$$B = -(1 - i) i J_3 + J_4 \left(1 + i_1\right) i_1 \quad (5)$$
$$C = J_2 + J_3 i^2 + J_4 i_1^2$$

$\delta\theta_1, \delta\theta_2, \delta\theta_3$ the virtual angular displacements.
From the kinematical analysis it results the relations:

$$\omega_3 = \omega_1 (1 - i) + \omega_2 i \quad (6)$$
$$\omega_4 = \omega_1 \left(1 + i_1\right) + \omega_2 i_1 . \quad (7)$$

The kinetic energy of the system is written as it follows:

$$E_C = \frac{1}{2}\left(A\dot{\theta}_1^2 + 2B\dot{\theta}_1\dot{\theta}_2 + C\dot{\theta}_2^2\right). \quad (8)$$

The virtual mechanical work is:

$$\delta L = M_1 \delta\theta_1 + M_2 \delta\theta_2 + M_3 \delta\theta_3 \quad (9)$$

and as the virtual displacements check the relation (6):

$$\delta\theta_3 = \delta\theta_1 (1 - i) + \delta\theta_2 i \quad (10)$$

it results

$$\delta L = \left[M_1 + M_2 (1 - i)\right]\delta\theta_1 + \left[M_2 + M_3 i\right]\delta\theta_2 \quad (11)$$

and from here are obtained the generalised forces:

$$Q_1 = M_1 + M_3 (1 - i); \; Q_2 = M_2 + M_3 i \quad (12)$$

By applying the Lagrange equations, one can obtain:

$$A\ddot{\theta}_1 + B\ddot{\theta}_2 = Q_1; B\ddot{\theta}_1 + C\ddot{\theta}_2 = Q_2 \quad (13)$$

or

$$\dot{\omega}_1 = \frac{\left[M_1 + M_3 (1 - i)\right]C + \left(M_2 + M_3 i\right)B}{AC - B^2}; \; \dot{\omega}_2 = \frac{\left[M_1 - M_3 (1 - i)\right]B + \left(M_2 + M_3 i\right)A}{AC - B^2}. \quad (14)$$

The integration of the movement equations (14) is made for the movement study in transient regime and for the movement stability study.

THE OBTAINING OF THE MECHANICAL MEASURES THROUGH THE ELEMENTS MODELLING

The determination on the *A, B, C* measures given by relation (5) involves the design of the mechanism, the determination of the elements masses and of their inertia moments. The most fast and efficient method is to model the elements using, and than to determine the mechanical measures. This way can be verified the mechanical stress and therefore can be checked the correctness of the design.

The problem that must be solved is the one of executing in a soft CAD the gears. In practice, the obtaining of the gears teeth is made through copying or rolling. The copying procedure contains: the casting, the stamping, the milling with disc cutters or with a finger, planing with a moulding tool etc. The rolling procedure is based on the engagement between the tool and the workpiece that is tooth shaped. The tool can be a cylindrical gear with zero teeth or a rack. Involute teeth profile of the gears is obtained most easily by rolling without slipping.

In [4] it is presented an obtaining method of the gears in AutoCAD using an AutoLisp function. In order to obtain the gears using the rolling without slipping procedure and the tool rack one may consider the algorithm:

I. A solid model (a perform article) is placed to approximate the gear in the position of 0°. "The semi-manufactured" from which is generated the gear is obtained from a cylinder using the AutoCAD "CYLINDER" command.
II. It is conveniently positioned in a good point, having height h and the outer radius $R_e = R + m + mx$, where: R – the radius of the division circle; m – the module; mx – the profile displacement.
III. It is placed a solid that materializes the generating rack.
IV. It is excluded from the solid which materializes the gear the solid that materializes the generating rack using the AutoCAD command AutoCAD "SUBTRACT" command.
V. The obtained solid is turned in a new position and the I ÷ III process is resumed again.

In order to obtain a gear after this algorithm one must perform at least 360 such operations (angular step of one degree). This is why the AutoLisp functions are needed to perform these operations with solids. Based on this algorithm and on an AutiLisp function, there have been realised the gears from the mechanical system. The process is so precise that it can be verified the correctness of the design of the gears, including the teeth gap and the possibility to attach satellites. The modelled gear in AutoCAD can be exported in other finite element analysis soft.

After the parts drawings of the items, it will be shape in AutoCAD the axes, the gins, the flanges and the gears using the algorithm proposed in [4]. Each piece will have its own name. To obtain the elements the assembly will be done taking into account table 1.

Table 1.

Element 1		Element 2		Element 3		Element 4	
Name	Piece	Name	Piece	Name	Piece	Name	Piece
Gin	1	Pinion $z = 35$	1	Gin	1	Pinion $z = 45$	3
Axle	1	Bearing 6205	2	Axle	1	Pinion $z = 30$	3
Bearing 6214	2	Gin	1	Bearing 6205	2	Satelite axle	3
Port-satelite flange	2	Axle	1	Pinion $z = 20$	1	Bearing 6004	6
Stub	1						

For the assembly of the elements in order to determine the mass and inertia moment, a new drawing is open where will be imported the constituent elements, according to the previous table. After the assembling, the element will be positioned according to the overall design.

In figure 3 is presented, in the left, the solid that generates element 1 (the axis of the element being OX, the origin being situated on the left face of the element), and in the right its photographic representation. To determine the geometrical and the mechanical properties of the solid it is used the AutoCAD "MASSPROP" command. After the correlation of the length and weight units it is obtained the mass of the element $m_1 = 12{,}6116$ Kg and the inertia moment in respect with the Ox axis, $J_1 = 0{,}0526365$ Kg·m^2.

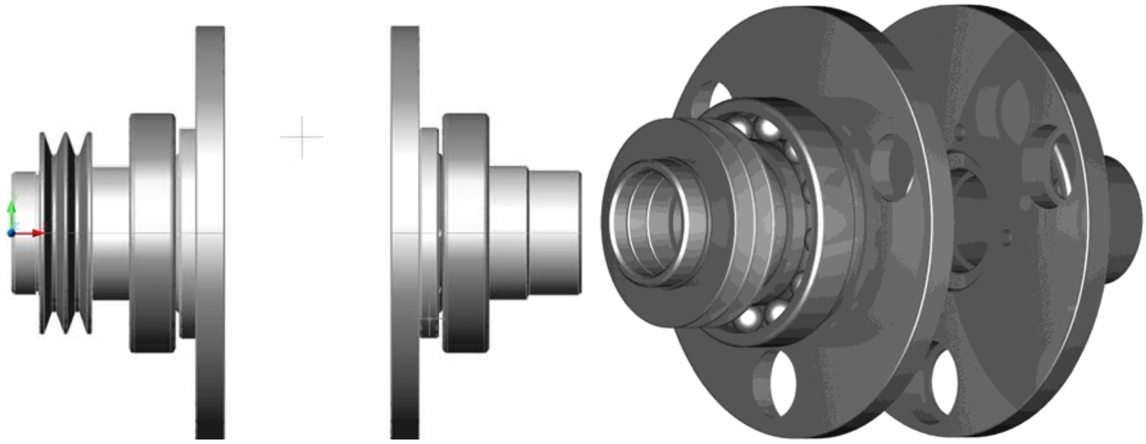

Fig. 3. The solid that generates element 1.

For element 2 one can obtain analogically the solid with the representations form figure 4, the axe of the element being Ox axis, the origin being situated on the left face of the element.

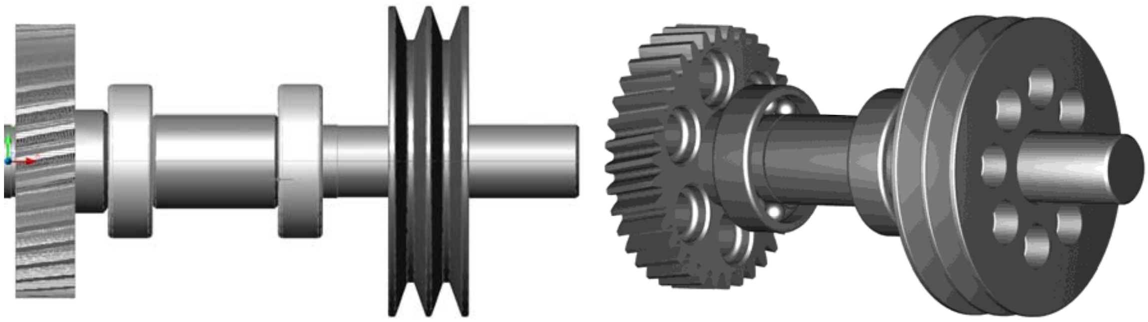

Fig. 4. The solid that generates element 2.

It is obtained the mass of the element $m_2 = 2{,}8085$ Kg, the inertia moment in respect with Ox axis $J_2 = 0{,}0023475$ Kg·m^2.

In figure 5 it is shown the representation of the element, its axis being Ox, and its origin being situated on the left face of the element.

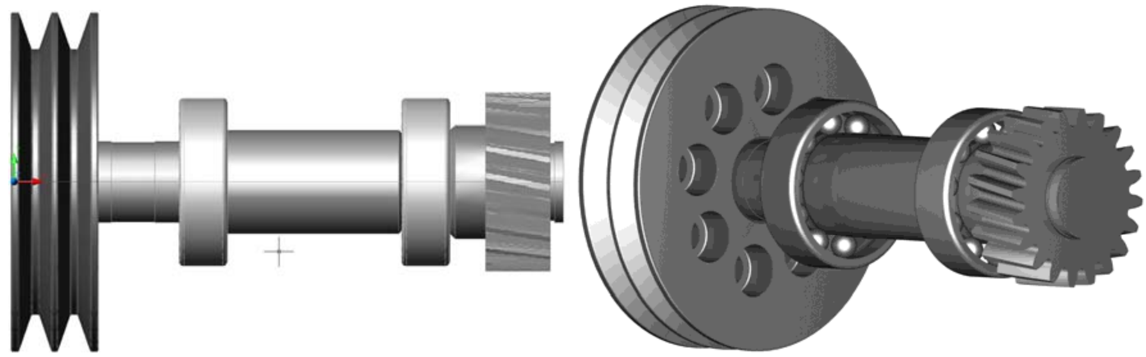

Fig. 5. The solid that generates element 3.

The weight of the element 3 is of $m_3 = 2{,}3699$ Kg and the inertia moment in respect with the Ox axis is $J_3 = 0{,}00175823$ Kg·m^2.

The solid that generates element 4 is represented in figure 6, its axis being Ox, and its origin being situated on the left face of the element.

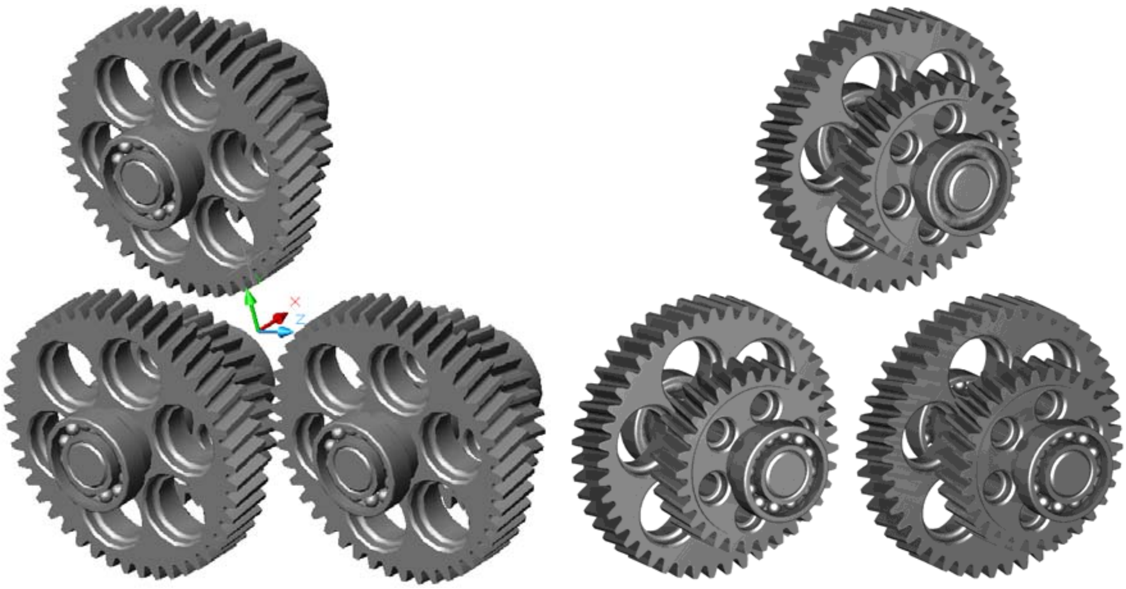

Fig. 6. The solid that generates element 4.

The weight of the element 4 is of $m_4 = 5{,}7439$ Kg and the inertia moment in respect with the Ox axis is $J_3 = 0{,}045242$ Kg·m^2.

Using these data, one can determine the A, B, C constants:

$A = 0{,}00689117$, $B = 0{,}1110185$, $C = 0{,}0760352$ (15)

NUMERICAL SIMULATION

To use the previous calculation formulas one will need to shape also the external characteristics of the engines. For the heat engine one can choose a polynomial of the 3rd grade to analytically approximate the characteristic $M(\omega)$. The characteristic given by the manufacturer (Honda) is represented in fig. 7.

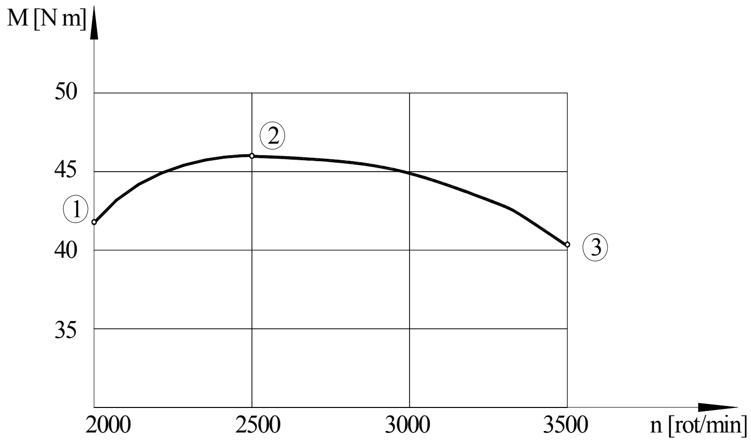

Fig. 7. The characteristic $M(\omega)$ of the heat engine.

It will be noted with M_1, M_2 and M_3 the moments corresponding to the angular speeds ω_1, ω_2 and ω_3, on the graphic from figure 7 these points have been noted with 1, 2 and 3.

The 3rd grade polynom that approximates the external characteristic is chosen under the following form:

$$M = a\omega^3 + b\omega^2 + c \quad (16)$$

In point 2, the moment reach its maximum and so the first order derivative is null:

$$\frac{dM_2}{d\omega_2} = 3a\omega^2 + 2b\omega = 0\,. \quad (17)$$

To determine the three constants a, b and the following conditions must be accomplished:

- in point 1 the moment has the known value M_1, the angular speed having the value ω_1, meaning:

$$M_1 = a\omega_1^3 + b\omega_1^2 + c\,, \quad (18)$$

- in point 2 the moment is maxim and the first order derivative is null:

$$3a\omega^2 + 2b\omega = 0\,, \quad (19)$$

- in point 3 the moment has the known value M_3, the angular speed having the value ω_3, meaning:

$$M_3 = a\omega_3^3 + b\omega_3^2 + c\,. \quad (20)$$

In this way was obtained a linear system of three equations with three unknowns from which we obtain the analytical expressions for the three unknowns:

$$a = \frac{M_1 - M_3}{\omega_1^3 - \omega_3^3 - 1.5\cdot\omega_2(\omega_1^2 - \omega_3^2)}\,; b = -1{,}5\cdot a\cdot\omega_2\,; c = M_1 - a\omega_1^3 - b\omega_1^2\,. \quad (21)$$

Knowing the values of the constants:

$$M_1 = 42\text{Nm}\,,\ \omega_1 = 2000\cdot\pi/30 = 209{,}44\,\text{rad/s},$$
$$M_2 = 46\,\text{Nm}, \omega_2 = 2500\cdot\pi/30 = 261{,}8\,\text{rad/s}, \quad (22)$$
$$M_3 = 41{,}5\,\text{Nm},\ \omega_3 = 3500\cdot\pi/30 = 366{,}52\,\text{rad/s},$$

there will result the following values for the constants a, b, c:

$$a = -1{,}10577\cdot10^{-7},\ b = 4{,}34\cdot10^{-5},\ c = 41. \quad (23)$$

In the case of the asynchronous electric engines, where the speed is adjusted with a frequency controlled electronic module, the external characteristic $M(\omega)$ indicated by some manufacturers specializing in this field is linear, during the common revolutions being even horizontal. In figure 8 is given the representation of the external characteristic of such a three-phased asynchronous engine.

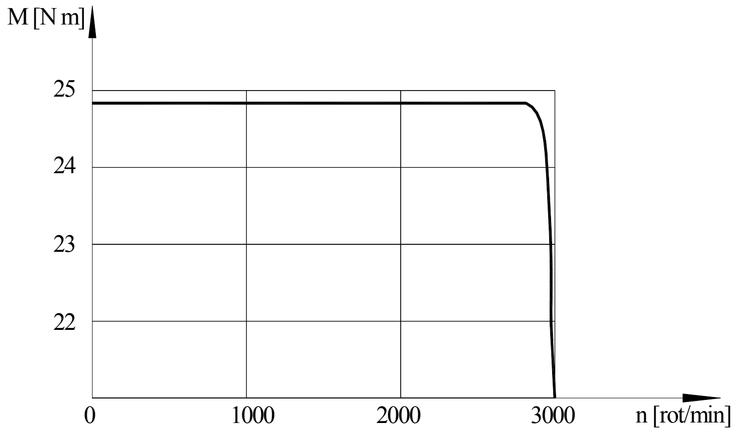

Fig. 8. The $M(\omega)$ characteristic of the frequency controlled asynchronous engine.

The maximum value of the moment given by the engine Type 132 S 38-2 is of 24,78 Nm and is practically constant in the scale 0 – 2000 rot/min.

For an alternator, the characteristic is the one from figure 9.

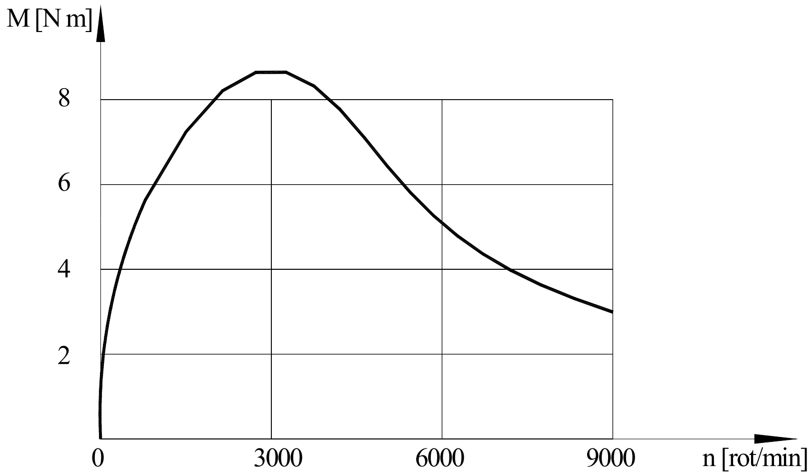

Fig. 9. The external characteristic of a 3 kW alternator.

Proceeding similarly and approximating the external characteristic with a polynomial of degree 3 for which are known:

$$M_1 = 9{,}44\text{Nm},\ \omega_1 = 1000\cdot\pi/30 = 104{,}716\,\text{rad/s},$$
$$M_2 = 5{,}44\,\text{Nm}, \omega_2 = 3000\cdot\pi/30 = 314{,}59\,\text{rad/s}, \tag{24}$$
$$M_3 = 4\,\text{Nm},\ \omega_3 = 7000\cdot\pi/30 = 733{,}04\,\text{rad/s},$$

for constants *a, b, c,* the values will result:

$$a = -3{,}068\cdot 10^{-8},\ b = 1{,}446\cdot 10^{-5},\ c = 8{,}31. \tag{25}$$

For the alternator to operate at nominal speed, it will be needed that its speed to be multiplied by the transmission belt with a ratio of $i_{alt} = 2$.

As a method of solving the differential equations of order 2 given by the relations (14) it is choose the Runge-Kutta method of order IV. The method involves transforming the system of two second order differential equations into four first order differential equations.
For this are used the notations :

$$\varphi_1 = \text{Y}_1,\ \varphi_2 = \text{Y}_2,\ \frac{\text{d}\varphi_1}{\text{d}t} = Y_3,\ \frac{\text{d}\varphi_2}{\text{d}t} = Y_4, \tag{26}$$

and equation (14) becomes:

$$\begin{aligned} \frac{\text{d}Y_3}{\text{d}t} &= \frac{[M_1 + M_3(1-i)]C + [M_2 + M_3 i - M_F]B}{AC - B^2}; \\ \frac{\text{d}Y_4}{\text{d}t} &= \frac{[M_1 + M_3(1-i)]B + [M_2 + M_3 i - M_F]A}{AC - B^2} \end{aligned} \tag{27}$$

The first order differential equations system will be:

$$\begin{cases} \dfrac{\text{d}Y_1}{\text{d}t} = Y_3 \\ \dfrac{\text{d}Y_2}{\text{d}t} = Y_4 \\ \dfrac{\text{d}Y_3}{\text{d}t} = \dfrac{[M_1 + M_3(1-i)]C + [M_2 + M_3 i - M_F]B}{AC - B^2} \\ \dfrac{\text{d}Y_4}{\text{d}t} = \dfrac{[M_1 + M_3(1-i)]B + [M_2 + M_3 i - M_F]A}{AC - B^2} \end{cases} \tag{28}$$

with initial conditions:

$$t = 0 \begin{cases} Y_1 = \varphi_{10} = 0 \\ Y_2 = \varphi_{20} = 0 \\ Y_3 = \omega_{10} = 0 \\ Y_4 = \omega_{20} = 0 \end{cases} . \tag{29}$$

It can thus pass to obtain numerical values using a computer program developed in Pascal language. The will be used some for simulation some of the cases experimentally analyzed to be able to make comparisons in the next chapter.

a) The mechanism driven by the electric engine (M_2), the MT heat engine (M_1), non-actionable and the generator (M_3) without any load. This is the start regime of a car with hybrid transmission. The torque generated by the electric engine M_2 is transmitted to the wheels of the car, a part of the power being transmitted to the mechanical coupling system of the power sources. The power transmitted to the heat engine is not enough to train it and start the engine, being sent to M_3. This, being charged, does not pass in generator regime, the speed being given by the relation:

$$\omega_3 = \omega_2 \cdot i \tag{30}$$

b) The mechanism driven by the heat engine MT (M_1), the electric engine (M_2) non-actionable and the generator (M_3) without any load. It is the case of an operating car in a medium regime, the power is transmitted to the wheel, while the generator operates with no sense (charged battery). To be able to obtain numerical values in this case it is needed to use the calculation program that allows solving the movement equations. The simulation is made for just some common operational values of the heat engine 2000, 2500 and 3000 rot/min. from the obtained characteristics can be noticed that the speeds of the three engines are equals, thing confirmed also by the experimental determinations. In figure 10 it is presented the variation of the revolutions when the heat engine operates at de 2000 rot/min.

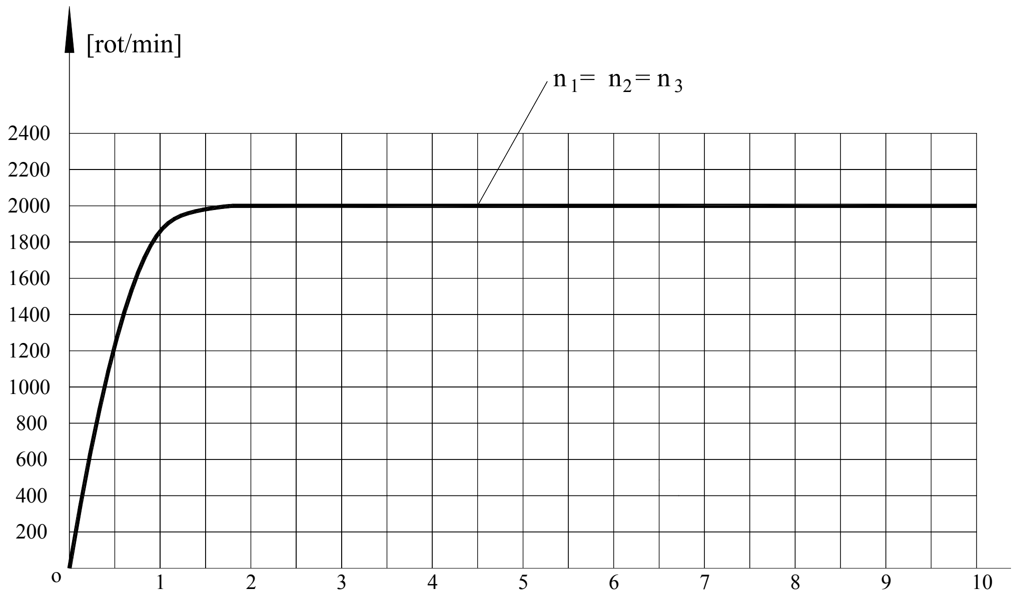

Fig. 10. Speed variation at a de 2000 rot/min operation of the heat engine.

CONCLUSIONS

In terms of theoretical research in the paper is presented a mathematical model of the mechanisms dynamics with two degrees of mobility. It allows to obtain numerical results using a computational program that solves the system of differential equations of motion. Necessary numerical values of the application are obtained by modeling the external characteristics of the engines and by modeling with solids the elements.

The mathematical model reflects the actual operational case of a car with hybrid drive system (heat engine, electric engine and electric generator). The calculation program compiled in Pascal programming language is flexible, allowing not only to obtain numerical results, but also the data transfer in a graphics construction specialized program.

Solid modeling is done in AutoCAD, it is very precise and gives the designer useful information. The obtained photographic representations are impressive. In fig. 11 is shown a photograph of the gear and the image obtained with AutoCAD by solid modeling of the elements after the drawings.

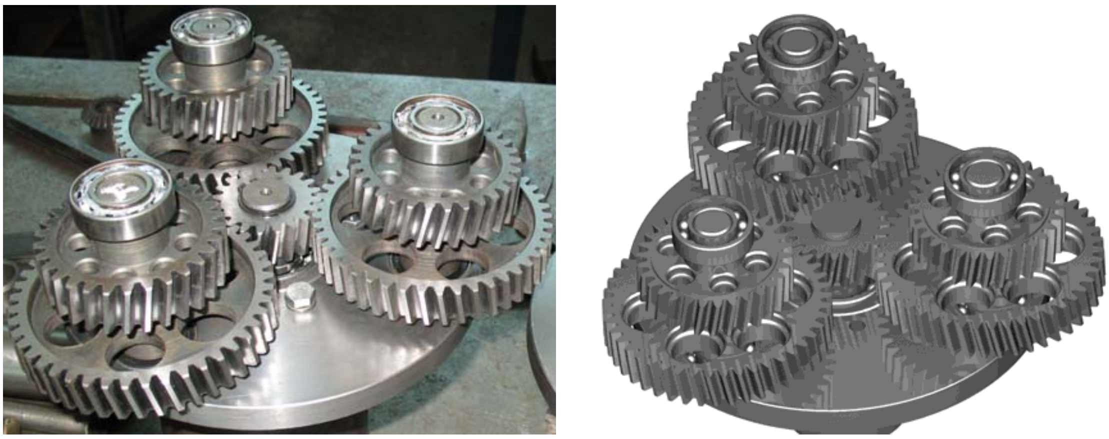

Fig. 11. Image obtained with the camera (left) and photographic representation with AutoCAD (right) of the gear.

Modeling with AutoCAD of all the mechanical system components has primarily a relief benefit for the determinations of the mechanical and inertia measures of the components and subassemblies. A second advantage is offered by the transfer models to the specialized software in the analysis of stresses and deformations.

Differences arising between the sizes determined with AutoCAD and those obtained by experimental measurements (weighing, experimental determination the of inertia moments, etc.) are due only to the lack of respect of the processing technology, modeling in AutoCAD being performed with great precision (double precision).

Modeling with AutoCAD of the gears is inspired by machining mechanical processes of the gears. The process is so precise that it can verify the correctness of the gears design, including the teeth gap and the possibility to mount satellites. The modeled gear in AutoCAD can be exported to other analysis software for finite element analysis.

For the frequent functioning cases were compared the experimentally obtained values with those obtained with the mathematical model, the differences obtained being less than 5%.

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