# DYNAMICS MODELING OF A POWER TRANSMISSION MECHANISM WITH LINKAGE AND INERTIAL MASS

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**Abstract:** This article describes construction of an model of inertial mechanism with two degrees of freedom. This mechanism is a mechanical application of the theory on the transmission of mechanical power by vibrations. The mechanical power is transmitted from engine to the output shaft through a system of oscillating levers, inertial masses end a unidirectional mechanism. This torque converter is analyzed by AMESim. Dynamics modeling of an automotive application will demonstrate its high performances characteristics. In the modeling of this power transmission system, the stiffness of the shaft and various control logics are included.

# **INTRODUCTION**

A mechanical linkage is an assembly of bodies connected to manage forces and movement. The movement of a body, or link, is studied using geometry so the link is considered to be rigid. The connections between links are modeled as providing ideal movement, pure rotation or sliding for example, and are called joints. A linkage modeled as a network of rigid links and ideal joints is called a kinematic chain. Linkages may be constructed from open chains, closed chains, or a combination of open and closed chains. Each link in a chain is connected by a joint to one or more other links. Thus, a kinematic chain can be modeled as a graph in which the links are paths and the joints are vertices, which is called a linkage graph.

Linkages are important components of machines and tools. Examples range from the four-bar linkage used to amplify force in a bolt cutter or to provide independent suspension in an automobile, to complex linkage systems in robotic arms and walking machines. The internal combustion engine uses a slider-crank four-bar linkage formed from piston, connecting rod and crankshaft to transform power from expanding burning gases into rotary power. Relatively simple linkages are often used to perform complicated tasks.

Mechanical linkages are usually designed to transform a given input force and movement into a desired output force and movement. The ratio of the output force to the input force is known as the mechanical advantage of the linkage, while the ratio of the input speed to the output speed is known as the speed ratio. The speed ratio and mechanical advantage are defined so they yield the same number in an ideal linkage.



Figure 1. Schematic representation of linkage mechanism

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The mechanism proposed to be analyzed in this article is presented in figure 1. It is composed of a skew-symmetric assembly of bars 1, 2, 3 and 4 connected through kinematic couples, A, B and C and a inertial mass located on the toolbar extension of bar 3.

# THE EQUATIONS OF MOTION

On the basis of the notations  $OA = l_1$ ;  $AB = l_2$ ;  $BC = l_3$ ;  $OC = l_4$ ;  $BD = l_5$ , can be write relations

$$x_A = l_1 \cos \varphi_1; \quad y_A = l_1 \sin \varphi_1 \tag{1}$$

$$x_C = l_4 \cos \varphi_4; \quad y_C = l_4 \sin \varphi_4 \tag{2}$$

$$x_B = x_A + l_2 \cos \varphi_2 = x_C + l_3 \cos \varphi_3 \tag{3}$$

$$y_B = y_A + l_2 \sin \varphi_2 = y_C + l_3 \sin \varphi_3 \tag{4}$$

where  $\varphi_1, \varphi_2, \varphi_3, \varphi_4$  are the position angles of bars 1, 2, 3 and 4 relative to the axis ox. Based on relations 3 and 4 we obtain

$$\varphi_2 = 2 \arctan \frac{S \pm \sqrt{S^2 - U^2 + C^2}}{C + U}$$
(5)

$$\varphi_3 = 2 \arctan \frac{S \pm \sqrt{S^2 - V^2 + C^2}}{C + V}$$
(6)

where

$$S = l_1 \sin \varphi_1 - l_4 \sin \varphi_4 \tag{7}$$

$$C = l_1 \cos \varphi_1 - l_4 \cos \varphi_4 \tag{8}$$

$$U = \frac{l_3^2 - l_2^2 - C^2 - S^2}{2l_2} \tag{9}$$

$$V = \frac{l_3^2 - l_2^2 + C^2 + S^2}{2l_3} \tag{10}$$

Based on the above relations, can be write

$$\varphi_2 = \varphi_2(\varphi_1, \varphi_4) \tag{11}$$

$$\varphi_3 = \varphi_3(\varphi_1, \varphi_4) \tag{12}$$

$$x_{D} = x_{B} + l_{5} \cos \varphi_{3} = x_{D}(\varphi_{1}, \varphi_{4})$$
(13)

$$y_D = y_B + l_5 \sin \varphi_3 = y_D(\varphi_1, \varphi_4)$$
 (14)

The kinetic energy of this mechanical system is

$$E = J_1 \frac{\dot{\varphi}_1^2}{2} + J_4 \frac{\dot{\varphi}_4^2}{2} + m_D \frac{\dot{x}_D^2 + \dot{y}_D^2}{2}$$
(15)

where  $J_1$ ,  $J_2$  are moments of inertia of the input and output shaft, and

$$\dot{\varphi}_{1} = \frac{d}{dt}\varphi_{1}; \ \dot{\varphi}_{4} = \frac{d}{dt}\varphi_{4}; \ \dot{x}_{D} = \frac{d}{dt}x_{D}; \ \dot{y}_{D} = \frac{d}{dt}y_{D}$$
(16)

The Lagrange equations for this mechanism can be write as form

$$\begin{cases} \frac{d}{dt} \left( \frac{\partial E}{\partial \dot{\varphi}_1} \right) - \frac{\partial E}{\partial \varphi_1} = T_1 \\ \frac{d}{dt} \left( \frac{\partial E}{\partial \dot{\varphi}_4} \right) - \frac{\partial E}{\partial \varphi_4} = -\operatorname{sign}(\varphi_4) T_4 \end{cases}$$
(17)

With notations  $y_1 = \varphi_1$ ;  $y_2 = \dot{\varphi}_1$ ;  $y_3 = \varphi_4$ ;  $y_4 = \dot{\varphi}_4$ , it is obtained the following relations



$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = f(y_1, y_2, y_3, y_4) \\ \dot{y}_3 = y_4 \\ \dot{y}_4 = g(y_1, y_2, y_3, y_4) \end{cases}$$
(18)

with initial conditions

$$t = 0; y_1(0) = y_2(0) = y_3(0) = y_4(0) = 0$$
 (19)

#### SIMULATION SCHEME

Simulation scheme of mechanical transmission is presented in figure 2. It is composed from input shaft, oscillating mechanism, unidirectional mechanism and output shaft. Simulation was made by two distinct way: 1 - by actuating with prime mover with constant speed  $\omega_1$ ; 2 - by actuating with prime mover with constant torque  $T_1$ .



Figure 4. Torques variation T<sub>4</sub> and T<sub>1</sub> depending on the output speed  $\omega_4$ 



Figure 5. Output power P<sub>4</sub> and efficiency  $\eta$  variation depending on the output speed  $\omega_4$ The main characteristics of operation by actualing with prime mover with constant speed are presented in figures 4,5 and 8a. The main characteristics of operation by actualing with prime mover with constant torque are presented in figures 6,7 and 8b. Time variation of load is presented in figure 3. Simulations was made with the following initial data:  $\omega_1 = 1500 rev/min$ ;  $T_1 = 20 Nm$ ;  $l_1 = 30 nm$ ;  $l_2 = 70 nm$ ;  $l_3 = 100 nm$ ;  $l_4 = 100 nm$ ;  $l_5 = 100 nm$ ;  $J_1 = 1kgm^2$ ;  $J_4 = 10kgm^2$ ;  $m_D = 5kg$ ;





Figure 8. Output speed  $\omega_4$  variation to square variation of output torque T<sub>4</sub> a) for prime mover with  $\omega_1 = const$ ; b) for prime mover with  $T_1 = const$ .

#### CONCLUSIONS

From the analysis of the above charts it is noticed major differences between operation with the loading in ascending sequence and the operation in descending sequence of loads. These differences





increase with increase of moments of inertia J1 and J2. The maximum power is transmitted to the median speed of the output shaft.

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