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The influence of the Technological Deviations over the Vibration Inherent Frequencies at Bending from the Three-Shafts Transmission

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Abstract. The technological deviations of manufacturing and assembly , lead to the change of the kinematic parameters and at further efforts in the kinematic pairs of the cardan joint mechanism from the component of the poly-cardan transmissions.The variation of these forces is constant in the local systems of reference of the kinematic pairs and harmonic varies in the general system of reference in which one of the axes coincides with the rotation axis.The component of the forces that acts perpendiculary on the direction this axis is a permanent source of excitation, leading to the change of their own frequencies and vibration modes at bending. In this paper are deducted the connections between technological deviations ,the excitation forces and own frequencies and vibration modes at bending and based on the results of numerical application conclusions will be drawn.

1. Introduction

Based on the dynamic model with distributed mass, in the previous papers [1]-[4] were studied the free vibrations of the two-shafts transmission with elastic frame, with and without technological deviations, were determined the own frequencies and were represented the vibration modes at bending.

Starting from the same model and the papers results [5]-[7] , the excitations that appear because of technical deviations will be stimulated and by using the mathematical model that I will establish in this paper will be determined the frequencies and inherent modes of vibration at bending of the three-shafts transmission.

2. Technological deviations in plücker coordinates

The cardan joint mechanism is a particular case of the 4r Symmetrical Spherical Quadrilateral Mechanism wchich being of third family it is multiple statically undetermined. Determination of the reactions from rotation kinematic pairs A, B, C, D is done by means of the linear elastic calculus using the method of the relative displacements with the expression of displacements in plücker coordinates [8].The kinematic pair from A, in the case of elastically linear calculation, it is considered fixed and the technological deviation of the AB element, represented in Figure 1, is given in the local reference system $Bxyz$, by the small rotation angle $\bar{\theta}_B^l$ and by the small displacement $\bar{\delta}_B^l = B\bar{B}'$.

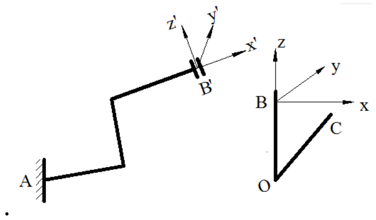

Figure 1. The technological deviation of the element AB

In plücker coordinates [4]-[7], [11] the deviations Δ_B^l, in the $Bxyz$ local reference system is written under the form:

$$\Delta_B^l = \left(\theta_{Bx}^l, \theta_{By}^l, \theta_{Bz}^l, \delta_{Bx}^l, \delta_{By}^l, \delta_{Bz}^l\right)^T. \tag{1}$$

where $\left(\theta_{Bx}^l, \theta_{By}^l, \theta_{Bz}^l, \delta_{Bx}^l, \delta_{By}^l, \delta_{Bz}^l\right)^T$ are the projections on the local axis of the vectors $\bar{\theta}_B^l, \bar{\delta}_B^l$.

3. The model with distributed masses for three-shafts transmission

The dynamic model is carried out on a three-shafts transmission used in the SUV field, whose construction model is shown in Figure 2.

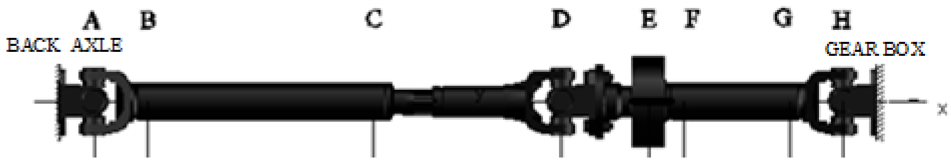

Figure 2. The Constructive model

The constructive solution of the three-shafts transmission is associated with the mechanical model presented in Figure 3. In the sections A, E and H are situated the elastic bearings with the elastic constants k_A, k_E, k_H and the harmonic excitation forces R_A, R_D, R_H of amplitude $\tilde{R}_A, \tilde{R}_E, \tilde{R}_H$ activating in the sections A, E, H.

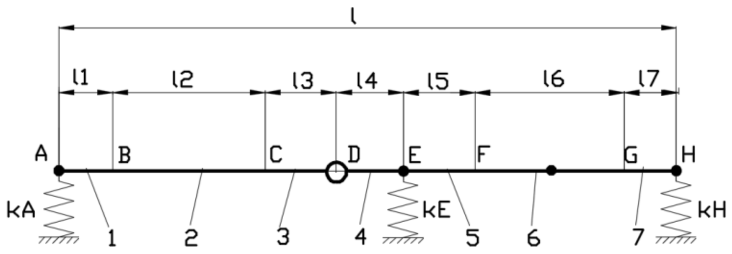

Figure 3. The equivalent mechanical model.

The dynamic modeling with distributed masses of the three-shafts transmission as well as the representation of the first two vibration modes at bending is presented in Figure 4.

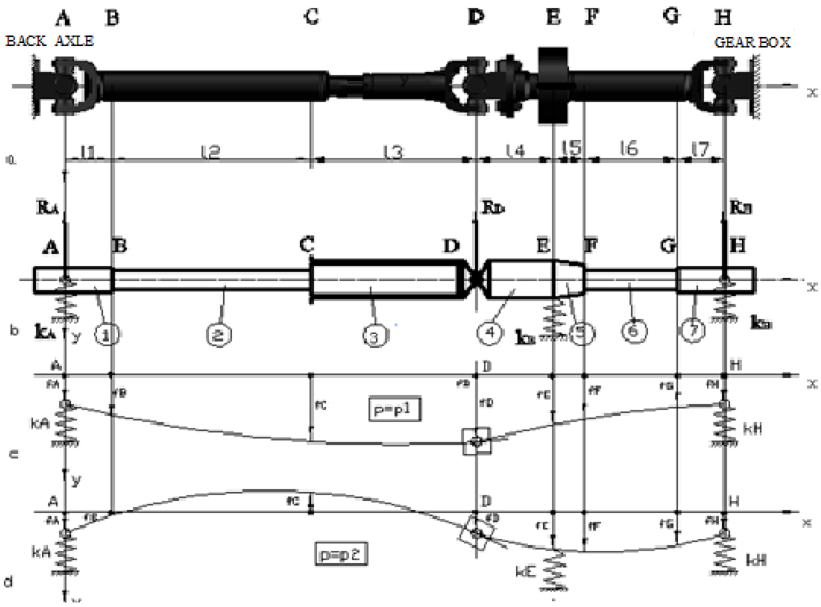

Figure 4. The dynamic modeling with distributed masses and first two vibration modes at bending

Next the notations are used [9]-[11]

a) f_i, θ_i, M_i, F_i - as the deflection, rotation, deflection moment and the cutting force from the current section

b) $\Delta_i\ ;\Delta_A\ ;\Delta_B\ ;\Delta_C\ ;\Delta_D\ ;\Delta_E\ ;\Delta_F\ ;\Delta_G\ ;\Delta_H$ - state vectors, defined by the relations:

$$\begin{aligned}
&\Delta_i = (f_i, \theta_i, M_i, F_i)^T\ \Delta_A = (f_A, \theta_A, M_A, F_A)^T\ \Delta_B = (f_B, \theta_B, M_B, F_B)^T \\
&\Delta_C = (f_C, \theta_C, M_C, F_C)^T\ \Delta_D = (f_D, \theta_D, M_D, F_D)^T . \Delta_E = (f_E, \theta_E, M_E, F_E)^T \\
&\Delta_F = (f_F, \theta_F, M_F, F_F)^T\ \Delta_G = (f_G, \theta_G, M_G, F_G)^T . \Delta_H = (f_H, \theta_H, M_H, F_H)^T
\end{aligned} \quad (2)$$

c) $x_i, \rho_i, A_i, E_i, I_{yi}$ - respectively, the length, density, area, transverse modulus of elasticity and geometrical moment of inertia mainly for the section corresponding to the index $i = 1,2,3,...7$

d) The parameters $\alpha_i ; z_i$ defined by the relations:

$$\alpha_i = \sqrt[4]{p^2 \frac{\rho_i A_i}{E_i I y i}} ; z_i = \alpha_i \cdot x_i . \quad (3)$$

where p is the vibration inherent pulse.

e) chz, shz - sin and cos the hyperbolic functions

$$ch(z_i) = \frac{e^{z_i} + e^{-z_i}}{2} \quad ; sh(z_i) = \frac{e^{z_i} - e^{-z_i}}{2} \quad . \quad (4)$$

f) $f_j(z_i)$, j=1,2,3,4 the Krâlov functions defined by relations:

$$f_1(z_i)=\frac{ch(z_i)+cos(z_i)}{2};\ f_2(z_i)=\frac{sh(z_i)+sin(z_i)}{2}$$
$$f_3(z_i)=\frac{ch(z_i)-cos(z_i)}{2};\ f_4(z_i)=\frac{sh(z_i)-sin(z_i)}{2}. \tag{5}$$

g) $F(z_i)$ – the Krâlov matrixes defined by relations:

$$F(z_i)=\begin{pmatrix} f_1(z_i) & f_2(z_i) & f_3(z_i) & f_4(z_i) \\ f_4(z_i) & f_1(z_i) & f_2(z_i) & f_3(z_i) \\ f_3(z_i) & f_4(z_i) & f_1(z_i) & f_2(z_i) \\ f_2(z_i) & f_3(z_i) & f_4(z_i) & f_1(z_i) \end{pmatrix}. \tag{6}$$

h) α , α^{-1} – for diagonal matrixes:

$$\alpha=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\alpha} & 0 & 0 \\ 0 & 0 & -\frac{1}{\alpha^2 EI_w} & 0 \\ 0 & 0 & 0 & -\frac{1}{\alpha^3 EI_w} \end{pmatrix};\alpha^{-1}=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & -\alpha^2 EI_w & 0 \\ 0 & 0 & 0 & -\alpha^3 EI_w \end{pmatrix}. \tag{7}$$

i) $R_i, i-1,2,3,...7$, the field matrixes of sections, [7]:

$$R_i=\alpha_i^{-1}\cdot F(\alpha_i x_i)\cdot\alpha_i. \tag{8}$$

By assimilating the bearings from sections A, D, H with the joints, results that

$$M_A=M_D=M_H=0 \tag{9}$$

and by also taking into account the elastic brackets results

$$F_A=k_A f_A; F_E^d=F_E^s+k_E f_E; F_H=-k_H f_H \tag{10}$$

In the absence of exciting force the obtained state vectors are:

$$\{\Delta_A\}=\begin{bmatrix}\tilde{f}_A\\ \tilde{\theta}_A\\ 0\\ k_A f_A\end{bmatrix};\{\Delta_D^s\}=\begin{bmatrix}\tilde{f}_D\\ \tilde{\theta}_D^s\\ 0\\ F_D\end{bmatrix};\{\Delta_D^d\}=\begin{bmatrix}\tilde{f}_D\\ \tilde{\theta}_D^d\\ 0\\ F_D\end{bmatrix};\{\Delta_E^s\}=\begin{bmatrix}\tilde{f}_E\\ \tilde{\theta}_E\\ 0\\ F_E^s\end{bmatrix};\{\Delta_E^d\}=\begin{bmatrix}\tilde{f}_E\\ \tilde{\theta}_E\\ 0\\ F_E^s+k_E f_E\end{bmatrix};\{\Delta_H\}=\begin{bmatrix}\tilde{f}_H\\ \tilde{\theta}_H\\ 0\\ -k_H f_H\end{bmatrix} \tag{11}$$

where the indices s,d refers to the left and right sections.
The equalities results

$$\begin{bmatrix}\tilde{f}_D\\ \tilde{\theta}_D^s\\ 0\\ F_D\end{bmatrix}=[R_3][R_2][R_1]\begin{bmatrix}\tilde{f}_A\\ \tilde{\theta}_A\\ 0\\ k_A f_A\end{bmatrix};\begin{bmatrix}\tilde{f}_E\\ \tilde{\theta}_E\\ M_E\\ F_E^s\end{bmatrix}=[R_4]\begin{bmatrix}\tilde{f}_D\\ \tilde{\theta}_D^d\\ 0\\ F_D\end{bmatrix};\begin{bmatrix}\tilde{f}_H\\ \tilde{\theta}_H\\ 0\\ -k_H f_H\end{bmatrix}=[R_7][R_6][R_5]\begin{bmatrix}\tilde{f}_E\\ \tilde{\theta}_E\\ M_E\\ F_E^s+k_E f_E\end{bmatrix} \tag{12}$$

that can be written under the form

$$[R_3][R_2][R_1]\begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 0\\ k_A & 1\end{bmatrix}\begin{bmatrix}f_A\\ \theta_A\end{bmatrix}-\begin{bmatrix}1 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}f_D\\ F_D\end{bmatrix}-\begin{bmatrix}0\\ 1\\ 0\\ 0\end{bmatrix}\theta_D^d=\{0\}$$

$$[R_4]\begin{bmatrix}1 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}f_D\\ F_D\end{bmatrix}-\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\tilde{f}_E\\ \tilde{\theta}_E\\ M_E\\ F_E^s\end{bmatrix}-\begin{bmatrix}0\\ 1\\ 0\\ 0\end{bmatrix}\theta_D^d=\{0\} \quad (13)$$

$$[R_7][R_6][R_5]\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\tilde{f}_E\\ \tilde{\theta}_E\\ M_E\\ F_E^s\end{bmatrix}-\begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 0\\ -k_H & 0\end{bmatrix}\begin{bmatrix}f_H\\ \theta_H\end{bmatrix}=\{0\}$$

By using the notations:

$$[E_1]=[R_3][R_2][R_1]\begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 0\\ k_A & 0\end{bmatrix};[E_2]=\begin{bmatrix}1 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix};[E_3]=\begin{bmatrix}0\\ 1\\ 0\\ 0\end{bmatrix};[E_4]=[R_4]\begin{bmatrix}1 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix}$$

$$[I_4]=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix};[E_5]=[R_7][R_6][R_5]\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ k_E & 0 & 0 & 1\end{bmatrix};[E_6]=\begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 0\\ -k_H & 0\end{bmatrix} \quad (14)$$

the equations (13), can be written under the form:

$$[E_1]\begin{bmatrix}f_A\\ \theta_A\end{bmatrix}-[E_2]\begin{bmatrix}f_D\\ F_D\end{bmatrix}-[E_3]\theta_D^s=\{0\}$$

$$[E_4]\begin{bmatrix}f_D\\ F_D\end{bmatrix}-[I_4]\begin{bmatrix}\tilde{f}_E\\ \tilde{\theta}_E\\ M_E\\ F_D^s\end{bmatrix}-[E_6]\theta_D^d=\{0\} \quad (15)$$

$$[E_5]\begin{bmatrix}\tilde{f}_E\\ \tilde{\theta}_E\\ M_E\\ F_D^s\end{bmatrix}-[E_6]\begin{bmatrix}f_H\\ \theta_H\end{bmatrix}=\{0\}$$

and with the notations:

$$\{\tilde{\Delta}\}=\left[f_A,\theta_A,f_D,\theta_D,\theta_D^s,\theta_D^d,f_E,\theta_E,M_E,F_D^s,f_H,\theta_H\right]^T \quad (16)$$

$$[E]=\begin{bmatrix}[E_1] & -[E_2] & -\{E_3\} & \{Q_{41}\} & [Q_{44}] & [Q_{42}]\\ [Q_{42}] & [E_4] & \{Q_{41}\} & -\{E_3\} & -[I_4] & [Q_{42}]\\ [Q_{12}] & [Q_{42}] & \{Q_{41}\} & \{Q_{41}\} & [E_5] & -[E_6]\end{bmatrix} \quad (17)$$

where by $[Q_{mn}]$ was noted the zero matrix with cu m lines and n columns, is obtained the homogeneous equation:

$$[E][\tilde{\Delta}]=\{0\} \quad (18)$$

that accepts the solution different of zero if:

$$det[E] = \{0\} \tag{19}$$

equation from which are determined the own pulsations.

In the case of harmonic excitations from A, D, H with the amplitudes $\tilde{R}_A, \tilde{R}_D, \tilde{R}_H$ in the equations (11) at the elements $k_A f_A$ is added $\tilde{R}_A$ at the element F_D added $\tilde{R}_D$ and at the element $-k_H f_H$ is added $-\tilde{R}_H$, and so the notations:

$$\{F_1\} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \tilde{R}_D \end{bmatrix} - [R_3][R_2][R_1]\begin{bmatrix} 0 \\ 0 \\ 0 \\ \tilde{R}_A \end{bmatrix}; \{F_2\} = -[R_4]\begin{bmatrix} 0 \\ 0 \\ 0 \\ \tilde{R}_D \end{bmatrix}; \{F_3\} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\tilde{R}_H \end{bmatrix} \tag{20}$$

$$\{\tilde{F}\} = \left[\{F_1\}^T, \{F_2\}^T, \{F_3\}^T\right]^T$$

it is obtained the matrix equation:

$$[E]\{\tilde{\Delta}\} = \{F\} \tag{21}$$

from which results the column matrix of amplitudes:

$$\{\tilde{\Delta}\} = [E]^{-1}\{\tilde{F}\} \tag{22}$$

4. Numerical application

Consider a mobile three-shafts transmission equipping a SUV vehicle whose construction model is shown in Figure 3, for the following construction features that are known and mechanical:

$$k_A = k_H = 85 \cdot 10^6 (N/m); k_E = 2 \cdot 10^6 (N/m);$$
$$l_1 = 0{,}07(m); l_2 = 0{,}59(m); l_3 = 0{,}25(m); l_4 = 0{,}12(m); l_5 = 0{,}05(m); l_6 = 0{,}36(m); l_7 = 0{,}07(m);$$
$$A_1 = A_3 = A_4 = A_5 = A_7 = 19{,}6 \cdot 10^{-4}(m^2); A_2 = A_6 = 4 \cdot 10^{-4}(m^2)$$
$$\rho_1 = \rho_2 = \rho_3 = \rho_4 = \rho_5 = \rho_6 = \rho_7 = 7800(kg/m^3).$$

Based on an algorithm that I will present in future work and a computer program developed in Excel or obtained first and second pulsation own value $p_1=155(s^{-1})$, $p_2=1072(s^{-1})$. Corresponding to this pulse graphs were drawn at the bending vibration inherent modes shown in Figure 5.

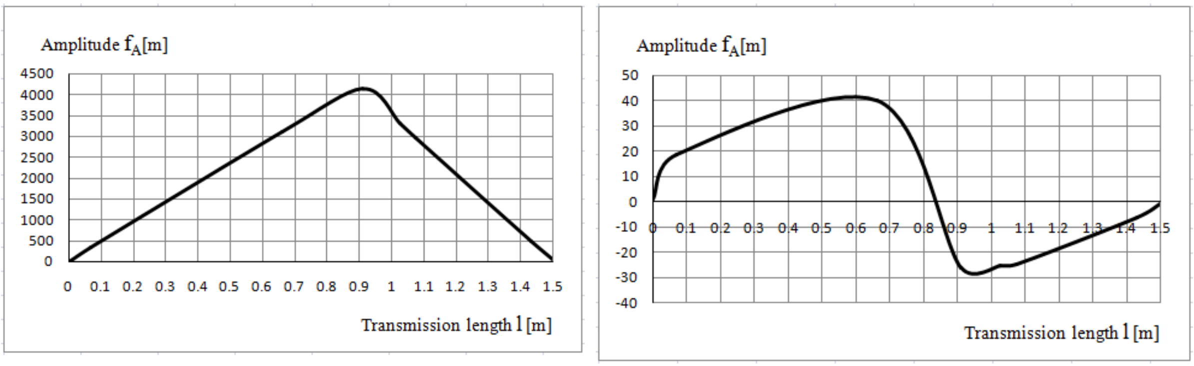

Figure 5. First and second pulsation own

5. Conclusions

The mathematical model presented, of the algorithm and of the program developed can be determinated the influence of each type of technological deviation an their own frequencies and vibration modes at bending of three-shafts transmissions.The matrix approach allows us to directly determine the own pulsations and ways of vibration of the three-cardan transmissions, the method could be extended also to poli-cardan transmission.

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