UNIVERSITY OF PITESTI
Faculty of Mechanics and Technology
SCIENTIFIC BULLETIN
AUTOMOTIVE series, year XXIV, no. 28

THE 11TH EDITION OF
The International Congress of Automotive and Transport Engineering
MOBILITY ENGINEERING AND ENVIRONMENT
November 8-10, 2017

Algorithm and numerical calculation of undetermined static reactions to the plan articulated quadrilateral mechanism with straight bars

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Article history
Received 10.07.2017
Accepted 24.09.2017

DOI https://doi.org/10.26825/bup.ar.2018.014

Abstract. This paper represents the numerical calculation of the undetermined static reactions to the planar plane quadrilateral mechanism with straight bars and kinematic rotation couplers.
The algorithm used was obtained using the relative displacement method and was presented in a previous paper. The method allows linear elastic calculation to determine undetermined static reactions.
For a numerical application a computational program was used, the Matlab software, with which the variation diagrams for the determined static reactions were obtained for the undetermined static reactions as well as the variations of the small angles of rotation in the kinematic couplers. The results obtained, we hope to be the ones they are looking for, and they are being developed
The work can be considered a novelty. The research conducted on the literature at both international and national levels has demonstrated the lack of such research concerns.
In conclusion, this work can be considered useful to researchers, manufacturers and users of such mechanisms. The quadrilateral mechanism is quite used, the results obtained in this paper will be useful to the researchers in this field, and the designers will more efficiently dimension the components of such a mechanism. The method may be useful in sizing calculations of existing vehicle mechanisms.

1. Introduction

In the previous work [4] a method for the calculation of undetermined static reactions was developed for the planar straight quadrilateral mechanism and kinematic rotation couplers

In the present paper, starting from the obtained analytical results, the algorithm and the calculation program for a numerical application will be elaborated.

2. Theoretical aspects. Remarks

The articulated quadrilateral mechanism is considered $ABCD$ from figure. 1,

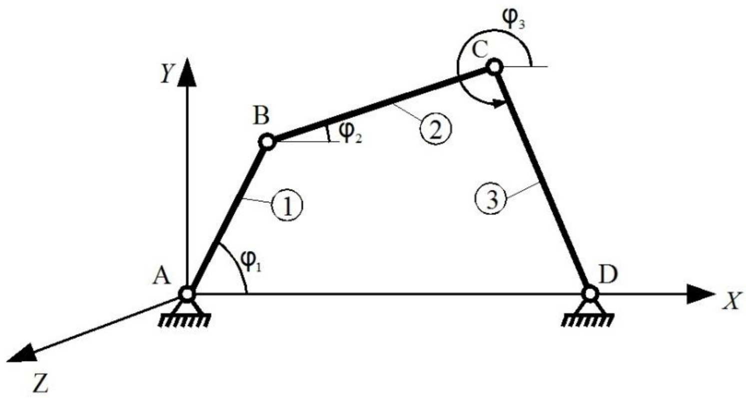

Figure. 1. The articulated quadrilateral mechanism

formed by the straight bars AB, BC, CD indexed by numbers $1,2,3$ and positioned through the angles $\varphi_1, \varphi_2, \varphi_3$ compared to the fixed reference system AXY and notations:

- R_{AX}, R_{AY} forces of reaction, statically determined from the joint A;
- M_{AZ} - momentum, statically determined from the joint A, moment that maintains equilibrium;
- R_{AZ} - reaction force, statically indeterminate from the joint A;
- M_{AX}, M_{AY} - moments of reaction, statically undetermined from the joint A;
- $\{R_A\}$ - column matrix of plűckeriene coordinates of the torsors reaction from A;
- $\{P_B, P_C, P_D\}$ - column matrices of plűckeriene coordinates torso's external forces acting in B, C, D;
- $\{U_B, U_C, U_D\}$ column matrices $[6],[8]$, attached to kinematic joints

$$\{U_B\} = [0 \quad 0 \quad 1 \quad Y_B \quad -Y_C \quad 0]^T;$$
$$\{U_C\} = [0 \quad 0 \quad 1 \quad Y_C \quad -Y_C \quad 0]^T; \tag{1}$$
$$\{U_D\} = [0 \quad 0 \quad 1 \quad Y_B \quad -Y_D \quad 0]^T$$

- $\{\tilde{U}_B\}$, $\{\tilde{U}_C\}$, $\{\tilde{U}_D\}$ column matrices given by relationships:

$$\{\tilde{U}_B\} = [Y_B \quad -X_B \quad 0 \quad 0 \quad 0 \quad 1]^T;$$
$$\{\tilde{U}_C\} = [Y_C \quad -X_C \quad 0 \quad 0 \quad 0 \quad 1]^T; \tag{2}$$
$$\{\tilde{U}_D\} = [Y_D \quad -X_C \quad 0 \quad 0 \quad 0 \quad 1]^T$$

- $[H_{AB}],[H_{BC}],[H_{CD}]$ the flexibility matrices of the bars AB, BC, CD;
- $[H_{AD}]$- the matrix given by relationship:

$$[H_{AD}] = [H_{AB}] + [H_{BC}] + [H_{CD}] \tag{3}$$

- $[K_{AD}]$- the stiffness matrix defined by the relationships

$$[K_{AD}] = [H_{AD}]^{-1} = \begin{bmatrix} 0 & 0 & K_{13} & K_{14} & K_{15} & 0 \\ 0 & 0 & K_{23} & K_{24} & K_{25} & 0 \\ K_{31} & K_{32} & 0 & 0 & 0 & K_{36} \\ K_{41} & K_{42} & 0 & 0 & 0 & K_{46} \\ K_{51} & K_{52} & 0 & 0 & 0 & K_{56} \\ 0 & 0 & K_{63} & K_{64} & K_{65} & 0 \end{bmatrix} \tag{4}$$

- $[K^1{}_{AD}],[K^2{}_{AD}]$ - matrices given by equality:

$$[K^1_{AD}] = \begin{bmatrix} K_{13} & K_{14} & K_{15} \\ K_{23} & K_{24} & K_{25} \\ K_{33} & K_{34} & K_{35} \end{bmatrix}, [K^2_{AD}] = \begin{bmatrix} K_{31} & K_{32} & K_{36} \\ K_{41} & K_{42} & K_{46} \\ K_{51} & K_{52} & K_{56} \end{bmatrix} \tag{5}$$

- $\{P_T\}$ - matrix column

$$\{P_T\} = -\begin{bmatrix} \{\tilde{U}_B\}^T \cdot \{P_B\} \\ \{\tilde{U}_C\}^T \cdot \{\{P_B\} + \{P_C\}\} \\ \{\tilde{U}_D\}^T \cdot \{\{P_B\} + \{P_C\} + \{P_D\}\} \end{bmatrix} \tag{6}$$

- $\{\tilde{\Delta}\}$ - matrix column

$$\{\tilde{\Delta}\} = [H_{BC}]\{P_B\} + [H_{CD}]\{\{P_B\} + \{P_C\}\} = [\tilde{\theta}_x \quad \tilde{\theta}_y \quad \tilde{\theta}_z \quad \tilde{\Delta}_x \quad \tilde{\Delta}_y \quad \tilde{\Delta}_z] \tag{7}$$

- $[A],[B]$ - matrices

$$[A] = \begin{bmatrix} Y_B & -X_B & 1 \\ Y_C & -X_C & 1 \\ Y_D & -X_D & 1 \end{bmatrix}; [B] = \begin{bmatrix} 1 & 1 & 1 \\ Y_B & Y_C & Y_D \\ -X_B & -X_C & -X_D \end{bmatrix} \tag{8}$$

- $\zeta_B, \zeta_C, \zeta_D$ - small angles of rotation in kinematic joins;
- $\{\zeta\}$ - matrix column.

$$\{\zeta\} = \{\zeta_B \quad \zeta_C \quad \zeta_D\}^T \tag{9}$$

With these notations [5], deduct the determined static reactions

$$\begin{bmatrix} R_{AX} \\ R_{AY} \\ M_{AZ} \end{bmatrix} = [A]^{-1}\{P_T\} \tag{10}$$

statically undetermined reactions

$$\begin{bmatrix} R_{AZ} \\ M_{AX} \\ M_{AY} \end{bmatrix} = -[K^2_{AD}] \begin{bmatrix} \tilde{\theta}_X \\ \tilde{\theta}_Y \\ \tilde{\Delta}_Z \end{bmatrix} \tag{11}$$

and the angles of rotation in the kinematic joints

$$\begin{bmatrix} \zeta_B \\ \zeta_C \\ \zeta_D \end{bmatrix} = [B]^{-1}[K^1_{AD}]^{-1}[A]^{-1}\{P_T\} + [B]^{-1} \begin{bmatrix} \tilde{\theta}_Z \\ \tilde{\theta}_X \\ \tilde{\Delta}_Y \end{bmatrix} \tag{12}$$

3. Calculation algorithm

It is considered as for bars indexed with $1,2,3$ the lengths are known l_i, areas A_i, of the normal sections, geometrical moments of inertia $I_{yi}, I_{zi}, I_{xi}\left(I_{xi} = I_{yi} + I_{zi}\right)$, elasticity modules E_i, G_i the flexibility matrices $[7]$,

$$[h_i] = \begin{bmatrix} 0 & 0 & 0_2 & \frac{l_i}{GI_{xi}} & 0 & 0 \\ 0 & 0 & \frac{l_i^2}{2E_iI_{yi}} & 0 & \frac{l_i}{E_iI_{yi}} & 0 \\ 0 & -\frac{l_i^2}{2EI_{zi}} & 0 & 0 & 0 & \frac{l_i}{E_iI_{zi}} \\ \frac{l_i}{EA_i} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{l_i^3}{3E_iI_{zi}} & 0 & 0 & 0 & -\frac{l^2{}_i}{2E_iI_{zi}} \\ 0 & 0 & \frac{l_i^3}{3EI_{yi}} & 0 & \frac{l_i^2}{2E_iI_{yi}} & 0 \end{bmatrix} \tag{13}$$

$i = 1,2,3$ and distance $AD = X_D$

Calculates for angle values φ_1 ranging from grade to grade:

a) the angles φ_3, φ_2 with relationships

$$A = 2l_3\left(X_D - l_! \cos\varphi_1\right); B = 2l_1l_3\sin\varphi_1; C = l_2^2 - X_D^2 - l_1^2 - l_3^2 + 2X_Dl_1\cos\varphi_1 \tag{14}$$

$$\varphi_3 = 2\pi + 2arctg\frac{B + \sqrt{A^2 + B^2 - C^2}}{C - A}; \varphi_2 = \arcsin\frac{l_3\sin\varphi_3 - l_1\sin\varphi_1}{l_2} \tag{15}$$

b) the coordinates of the points B, C, D;

$$X_B = l_1\cos\varphi_1; Y_B = l_1\sin\varphi_1; X_C = X_B + l_2\cos\varphi_2; Y_C = Y_B + l_2\sin\varphi_2; Y_D = 0 \tag{16}$$

c) translation matrices;

$$[G_1] = [0]; [G_2] = \begin{bmatrix} 0 & 0 & Y_B \\ 0 & 0 & -X_B \\ -Y_B & X_B & 0 \end{bmatrix}; [G_3] = \begin{bmatrix} 0 & 0 & Y_C \\ 0 & 0 & -X_C \\ -Y_C & X_C & 0 \end{bmatrix} \tag{17}$$

d) rotation matrices;

$$[R_i] = \begin{bmatrix} \cos\varphi_i & -\sin\varphi_i & 0 \\ \sin\varphi_i & \cos\varphi_i & 0 \\ 0 & 0 & 1 \end{bmatrix}; i = 1,2,3 \tag{18}$$

e) position matrices;

$$[T_i] = \begin{bmatrix} [R_i] & [0] \\ [G_i][R_i] & [R_i] \end{bmatrix}; [T_i]^{-1} = \begin{bmatrix} [R_i]^T & [0] \\ [R_i]^T[G_i]^T & [R_i]^T \end{bmatrix}; i = 1,2,3 \tag{19}$$

f) the flexibility matrices;

$$[H_i] = [T_i][k_i][T_i]^{-1}; [H_{AB}] = [H_1]; [H_{BC}] = [H_2]; [H_{CD}] = [H_3] \tag{20}$$

g) the total flexibility matrix

$$[H_{AD}] = [H_{AB}] + [H_{BC}] + [H_{CD}] \tag{21}$$

h) matrices $[K_{AD}], [K^1_{AD}], [K^2_{AD}]$ with relationships (3), (4), (5);

i) column matrices $[P_T], [\tilde{\Delta}]$ with relationships (6), (7);

j) the matrices $[A], [B]$ with relationships (8);

k) static reactions determined by the relationship (10);

l) static reactions not determined with the relationship(11);

m) the rotation of the kinematic couple with the relationship(12);.

4. Numerical application

It determines statically the reactions, determined and indetermined and then the angular rotation of the couplings for the mechanism of Figure 1, knowing that the element 3 is actuated in D in the direction of the axis AZ, for a moment $\tilde{M}$ and knowing it as punctual C acting force P in the direction of the axis AZ. It is considered that the bars have equal, circular cross sections of diameter d.

Numerical data:

$l_1 = l = 0.2\,m; l_2 = l_3 = 3*l; X_D = 3*l; d = 0.01\,m; E_i = E = 2.1\cdot 10^{11} N\cdot m^{-2}; G_1 = G = 0.8\cdot 10^{11} N\cdot m^{-2};$

$I_{zi} = I_{yi} = \frac{\pi d^4}{64}; I_{xi} = \frac{\pi d^4}{32}; \{P_B\} = \{0\}; \{P_C\} = [0 \quad 0 \quad P \quad PY_c \quad -PX_c \quad 0]^T; \{P_D\} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \tilde{M}]^T;$

$P = 100\,N; \tilde{M} = 20\,N\cdot m$

In the numerical case the calculation program is elaborated using the Matlab code, on the basis of which are represented the following diagrams

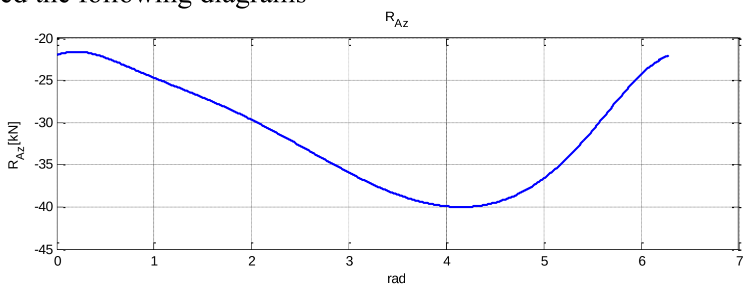

Figure. 2. The statically undetermined reactions R_{AZ} vs. φ_1

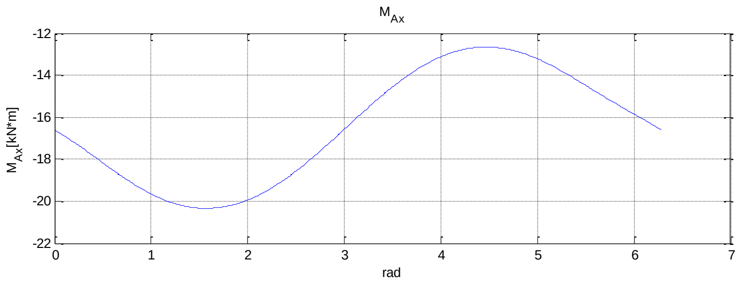

Figure. 3. The statically undetermined reactions M_{AX} vs. φ_1

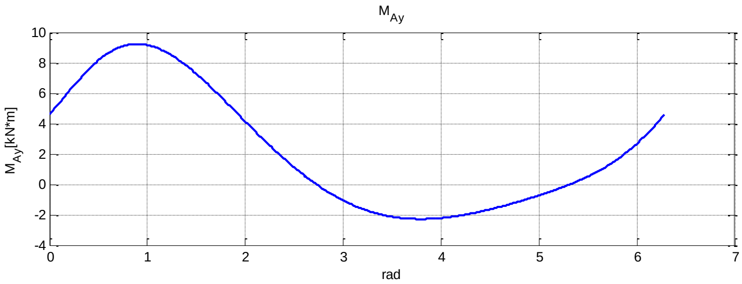

Figure. 4. The statically undetermined reactions M_{AY} vs. φ_1

5. Conclusion

Under these conditions, based on the calculations described in the paper, a computational program was drawn using the Matlab code, and the results obtained for the undetermined static reactions are transcribed in the diagrams of Fig. 2-4. In the absence of motion-compatible external forces (forces in the plane of motion and moment perpendicular to the plane of motion), the components R_{AX}, R_{AY}, M_{AZ} compatible with the movement of the reaction from A are null. The obtained variation diagrams represent periodical variations. For R_{Az} the extreme, minimum and maximum values are between; -40.0089, -21.6020. For component M_{AX}, values are between -20.3390 and -12.6537 and for M_{AY} between -2.2803 and 9.2389. Static undetermined reaction values are significant and can influence the movement of the mechanism. The minimum reaction moment, M_{AX}, in the absolute value, is greater than the momentum of the drive.

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