



The determination of the mechanical friction work in a cam – follower couple of complex cam mechanisms. Part II - numerical application

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Article history

Received 15.08.2020

Accepted 20.09.2020

DOI <https://doi.org/10.26825/bup.ar.2020.012>

Abstract: This paper is a continuation of a previous paper, where we presented the analytical relations that lead to obtaining the mechanical friction work from the cam-follower couple of a complex cam mechanism. The obtained results are presented as diagrams obtained by a computer program.

Keyword: cam, follower, positions, velocities, reactions, mechanical work

1. Introduction

Complex cam mechanisms are often used in the technical field. In a previous paper, we presented the dynamic and kinematic aspects and also the expression of the mechanical friction work from the cam-follower couple. The present paper is giving an application about a quadrilateral articulated mechanism, where the cam actuates the connecting rod of the mechanism that is attached to a round grooved follower. There are presented, in order, the calculation algorithms for: kinematic analysis, dynamical analysis and the results obtained by the numerical application.

2. The kinematic technical diagram of the mechanism

In fig.1 we are presenting the technical diagram of a complex cam mechanism. The cam 1 actuates the connecting rod 2 of the quadrilateral articulated mechanism $OABC$. The cam is circular, of radius r_1 , and has the base circle radius r_0 . On the connecting rod 2 is mounted the circular follower of radius R . There are also known: the lengths of the mechanism elements: l_{OA} , l_{AB} , l_{AD} , l_{BC} , the positions of the couples at the base $O(x_O, y_O)$, $C(x_C, y_C)$, $O_1(x_{O_1}, y_{O_1})$, the position of point $E(x_E, y_E)$.

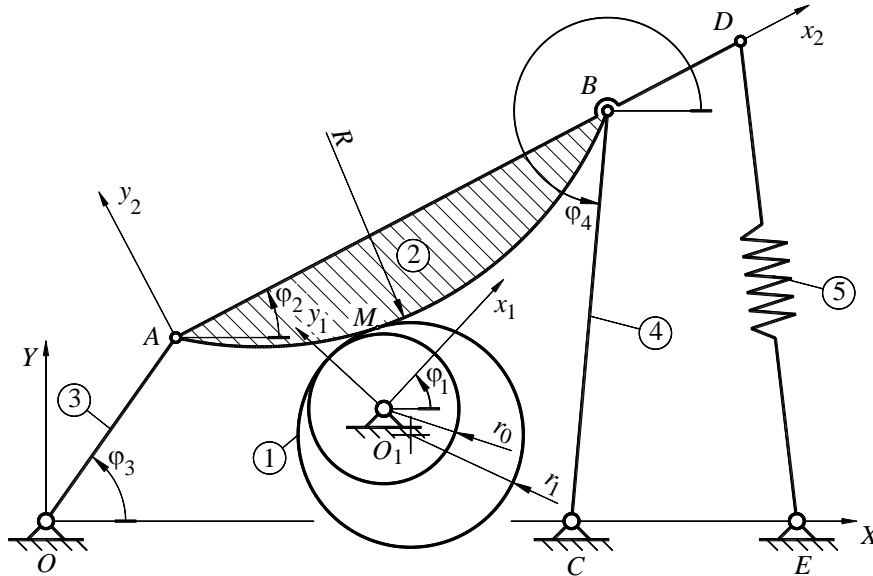


Figure 1. Complex cam mechanism.

3. The kinematic analysis of the mechanism

The profiles of the cam and the follower are given by the parametrical equations:

$$\begin{aligned} x_1 &= x_1(\lambda_1); y_1 = y_1(\lambda_1), \\ x_2 &= x_2(\lambda_2); y_2 = y_2(\lambda_2). \end{aligned} \quad (1)$$

We will first determine the reference position of the mechanism (when the cam is on the base circle). In fig. 2, the position angles of the elements are noted with the index 0.

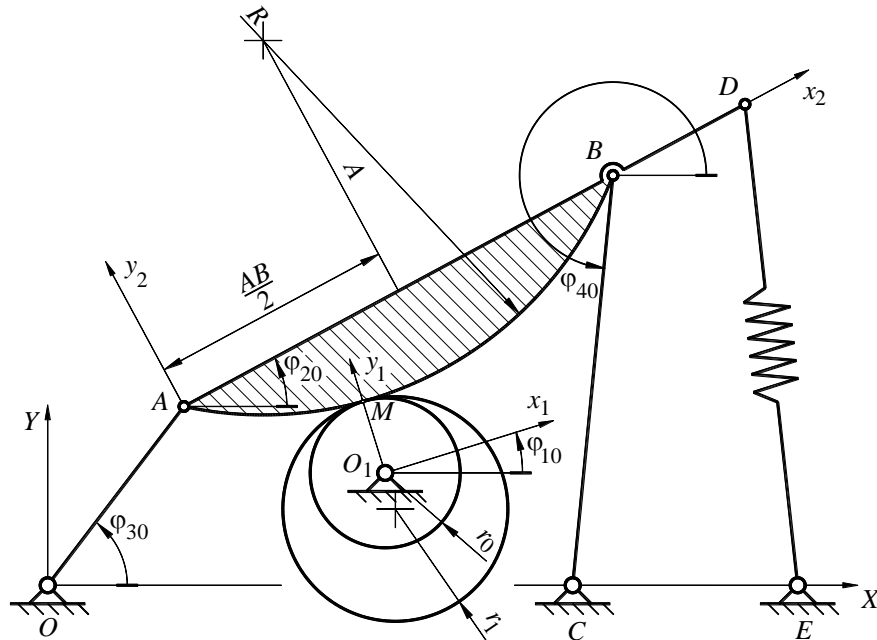


Figure 2. The positioning of the mechanism with the cam on the base circle.

For determining the position of the cam and the elements of the quadrilateral articulated mechanism, we project the contour vector relations on the OX and OY axis system:

$$\begin{aligned} \overline{OA} + \overline{AB} + \overline{BC} &= \overline{OC}, \\ \overline{OA} + \overline{AO_1} &= \overline{OO_1}, \end{aligned} \quad (2)$$

where it is added the tangency condition of the two profiles in the point M .

It will result a non-linearly system of five equations with five unknowns:

$$\begin{cases} F_1 = l_{OA} \cos \varphi_{30} + l_{AB} \cos \varphi_{20} + l_{BC} \cos \varphi_{40} - x_C + x_O = 0 \\ F_2 = l_{OA} \sin \varphi_{30} + l_{AB} \sin \varphi_{20} + l_{BC} \sin \varphi_{40} - y_C + y_O = 0 \\ F_3 = l_{OA} \cos \varphi_{30} + x_{20} \cos \varphi_{20} - y_{20} \sin \varphi_{20} - x_{O_1} + r_0 \sin \varphi_{10} = 0 \\ F_4 = l_{OA} \sin \varphi_{30} + x_{20} \sin \varphi_{20} + y_{20} \cos \varphi_{20} - y_{O_1} - r_0 \cos \varphi_{10} = 0 \\ F_5 = x'_2 \sin(\varphi_{20} - \varphi_{10}) + y'_2 \cos(\varphi_{20} - \varphi_{10}) = 0 \end{cases} \quad (3)$$

Noting with $\{F\}$ the previously determined functions vector:

$$\{F\} = (F_1 \quad F_2 \quad F_3 \quad F_4 \quad F_5)^T \quad (4)$$

and with $\{\Phi\}$ the vector of the unknowns:

$$\{\Phi\} = (\varphi_{30} \quad \varphi_{20} \quad \varphi_{40} \quad \varphi_{10} \quad \lambda_{20})^T. \quad (5)$$

The system (3) is written under the form:

$$\{F\} = 0, \quad (6)$$

and it is solved with the Newton-Raphson method. We are starting from an approximate value:

$$\{\Phi^{(1)}\} = (\varphi_{30}^{(1)} \quad \varphi_{20}^{(1)} \quad \varphi_{40}^{(1)} \quad \varphi_{10}^{(1)} \quad \lambda_{20}^{(1)})^T \quad (7)$$

and after the algorithm:

$$\{\Phi\}^{(i)} = \{\Phi\}^{(i-1)} - [W]_{i-1}^{-1} \{F\}^{(i-1)} \quad (8)$$

after i steps, the solution is determined with the precision ε :

$$|\{\Phi\}^{(i)} - \{\Phi\}^{(i-1)}| \leq \varepsilon. \quad (9)$$

In (8) was noted with $[W]$ the Jacobean of the system (6). In the case of a numerical application we are considering sizes: $l_{OA} = 0,030$ m, $l_{AB} = 0,065$ m, $l_{BC} = 0,070$ m, $l_{AD} = 0,085$ m, $x_O = 0$, $y_O = 0$, $x_C = 0,070$ m, $y_C = 0$, $x_E = 0,100$ m, $y_E = 0$, $x_{O_1} = 0,045$ m, $y_{O_1} = 0,015$ m, $r_0 = 0,010$ m, $r_1 = 0,015$ m, $R = 0,050$ m, $A = 0,038$ m, the parametrical equations of the cam: $x_1 = r \sin \lambda_1$, $y_1 = r \cos \lambda_1 - (r - r_0)$, the parametrical equations of the follower (the position of the center of the circle being on the perpendicular constructed from the middle of AB at the distance A):

$$x_2 = \frac{AB}{2} + R \sin \lambda_2, \quad y_2 = -(R \cos \lambda_1 - A).$$

The analytical expressions of the first and second order derivatives of the parametric equations of the cam and follower are given by the relations:

$$\begin{aligned} x'_1 &= r \cos \lambda_1, \quad y'_1 = -r \sin \lambda_1, \quad x''_1 = -r \sin \lambda_1, \quad y''_1 = -r \cos \lambda_1, \\ x'_2 &= R \cos \lambda_2, \quad x''_2 = -R \sin \lambda_2, \quad x'''_2 = -R \sin \lambda_2, \quad y''_2 = R \cos \lambda_2. \end{aligned} \quad (10)$$

Based on the previous relations, a calculation program is made.

Starting from the approximate values at the first iteration:

$$\{\Phi^{(1)}\} = (45^\circ \quad 25^\circ \quad 270^\circ \quad 30^\circ \quad 0)^T \quad (11)$$

the exact values are obtained for a better precision of determining the solution $\varepsilon = 10^{-6}$:

$$\{\Phi\} = (52,78096^\circ \quad 28,345212^\circ \quad 264,3964^\circ \quad 15,738287^\circ \quad -12,6069^\circ)^T \quad (12)$$

The relations (2) and the tangency condition of the cam with the follower in the contact point, are valid for any position of the mechanism.

In a random position of the mechanism, the vector (3) of the five functions $\{F\}$ is:

$$\left\{ \begin{array}{l} F_1 = l_{OA} \cos(\varphi_{30} + \varphi_3) + l_{AB} \cos(\varphi_{20} + \varphi_2) + l_{BC} \cos(\varphi_{40} + \varphi_4) - x_C + x_O = 0 \\ F_2 = l_{OA} \sin(\varphi_{30} + \varphi_3) + l_{AB} \sin(\varphi_{20} + \varphi_2) + l_{BC} \sin(\varphi_{40} + \varphi_4) - y_C + y_O = 0 \\ F_3 = l_{OA} \cos(\varphi_{30} + \varphi_3) + x_2 \cos(\varphi_{20} + \varphi_2) - y_2 \sin(\varphi_{20} + \varphi_2) - x_{O_1} - \\ \quad - x_1 \cos(\varphi_{10} + \varphi_1) + y_1 \sin(\varphi_{10} + \varphi_1) = 0 \\ F_4 = l_{OA} \sin(\varphi_{30} + \varphi_3) + x_2 \sin(\varphi_{20} + \varphi_2) + y_2 \cos(\varphi_{20} + \varphi_2) - y_{O_1} - \\ \quad - x_1 \sin(\varphi_{10} + \varphi_1) - y_1 \cos(\varphi_{10} + \varphi_1) = 0 \\ F_5 = (x'_1 x'_2 + y'_1 y'_2) \sin(\varphi_{20} - \varphi_{10} + \varphi_2 - \varphi_1) - (x'_1 y'_2 - x'_2 y'_1) \cos(\varphi_{20} - \varphi_{10} + \varphi_2 - \varphi_1) = 0 \end{array} \right. \quad (13)$$

and the vector $\{\varphi\}$ of the unknowns:

$$\{\Phi\} = (\varphi_3 \quad \varphi_2 \quad \varphi_4 \quad \lambda_1 \quad \lambda_2)^T. \quad (14)$$

The Jacobean $[W]$ of the system has the components:

$$\begin{aligned} \frac{\partial F_1}{\partial \varphi_3} &= -l_{OA} \sin(\varphi_{30} - \varphi_3), \quad \frac{\partial F_1}{\partial \varphi_2} = -l_{AB} \sin(\varphi_{20} - \varphi_2), \quad \frac{\partial F_1}{\partial \varphi_4} = -l_{BC} \sin(\varphi_{40} - \varphi_4), \quad \frac{\partial F_1}{\partial \lambda_1} = 0, \quad \frac{\partial F_1}{\partial \lambda_2} = 0; \\ \frac{\partial F_2}{\partial \varphi_3} &= l_{OA} \cos(\varphi_{30} - \varphi_3), \quad \frac{\partial F_2}{\partial \varphi_2} = l_{AB} \cos(\varphi_{20} - \varphi_2), \quad \frac{\partial F_2}{\partial \varphi_4} = l_{BC} \cos(\varphi_{40} - \varphi_4), \quad \frac{\partial F_2}{\partial \lambda_1} = 0, \quad \frac{\partial F_2}{\partial \lambda_2} = 0; \\ \frac{\partial F_3}{\partial \varphi_3} &= -l_{OA} \sin(\varphi_{30} - \varphi_3), \quad \frac{\partial F_3}{\partial \varphi_2} = -x_2 \sin(\varphi_{20} + \varphi) - y_2 \cos(\varphi_{20} + \varphi), \quad \frac{\partial F_3}{\partial \varphi_4} = 0, \\ &\quad \frac{\partial F_3}{\partial \lambda_1} = -x'_1 \cos(\varphi_{10} + \varphi_1) + y'_1 \sin(\varphi_{10} + \varphi_1), \quad \frac{\partial F_3}{\partial \lambda_2} = 0; \\ \frac{\partial F_4}{\partial \varphi_3} &= l_{OA} \cos(\varphi_{30} - \varphi_3), \quad \frac{\partial F_4}{\partial \varphi_2} = x_2 \cos(\varphi_{20} + \varphi) - y_2 \sin(\varphi_{20} + \varphi), \quad \frac{\partial F_4}{\partial \varphi_4} = 0, \\ &\quad \frac{\partial F_4}{\partial \lambda_1} = -x'_1 \sin(\varphi_{10} + \varphi_1) - y'_1 \cos(\varphi_{10} + \varphi_1), \quad \frac{\partial F_4}{\partial \lambda_2} = x'_2 \sin(\varphi_{20} + \varphi_2) + y'_2 \cos(\varphi_{20} + \varphi_2); \\ \frac{\partial F_5}{\partial \varphi_3} &= 0, \quad \frac{\partial F_5}{\partial \varphi_2} = (x'_1 x'_2 + y'_1 y'_2) \cos(\varphi_{20} - \varphi_{10} + \varphi_2 - \varphi_1) + (x'_1 y'_2 - x'_2 y'_1) \sin(\varphi_{20} - \varphi_{10} + \varphi_2 - \varphi_1), \\ \frac{\partial F_5}{\partial \varphi_4} &= 0, \quad \frac{\partial F_5}{\partial \lambda_1} = (x''_1 x''_2 + y''_1 y''_2) \sin(\varphi_{20} - \varphi_{10} + \varphi_2 - \varphi_1) - (x''_1 y''_2 - x''_2 y''_1) \cos(\varphi_{20} - \varphi_{10} + \varphi_2 - \varphi_1), \\ &\quad \frac{\partial F_5}{\partial \lambda_2} = (x'_1 x''_2 + y'_1 y''_2) \sin(\varphi_{20} - \varphi_{10} + \varphi_2 - \varphi_1) - (x'_1 y''_2 - x''_2 y'_1) \cos(\varphi_{20} - \varphi_{10} + \varphi_2 - \varphi_1); \end{aligned} \quad (15)$$

Analogous to the determination of the reference position of the mechanism, the Newton-Raphson algorithm is used to determine the exact solution in step i :

$$\{\Phi\}^{(i)} = \{\Phi\}^{(i-1)} - [W]_{i-1}^{-1} \{F\}^{(i-1)} \quad (16)$$

The iterative process continues until all the solutions in step i meet the condition:

$$\left| \Phi_j^{(i)} - \Phi_j^{(i-1)} \right| \leq \varepsilon, \quad j = 1, 2, 3, 4, 5. \quad (17)$$

where with ε was noted the maximum value that determines the solution.

The calculation program is completed with a repetitive (FOR) cycle, where the angle φ_1 is being given values in the interval $[0^\circ, \dots, 360^\circ]$, with a constant angular step of $\Delta\varphi_1 = 1^\circ$. For each step, determination will be done in a repetitive cycle (REPEAT - UNTIL), the values being given by the relations (13), (15) and (16) with the precision $\varepsilon = 10^{-6}$. The starting values of the iterative process: $\varphi_3^{(0)} = 0^\circ$, $\varphi_2^{(0)} = 0^\circ$, $\varphi_4^{(0)} = 0^\circ$, $\lambda_1^{(0)} = 0^\circ$, $\lambda_2^{(0)} = 0^\circ$, are only necessary for the first step ($\varphi_1 = 0$), for the next steps the approximate values are the exact values determined in the previous step. The values obtained at the exit of the repetitive cycle are stored as 360-position vectors and are written to files, on the basis of which are obtained the diagrams of fig. 3.

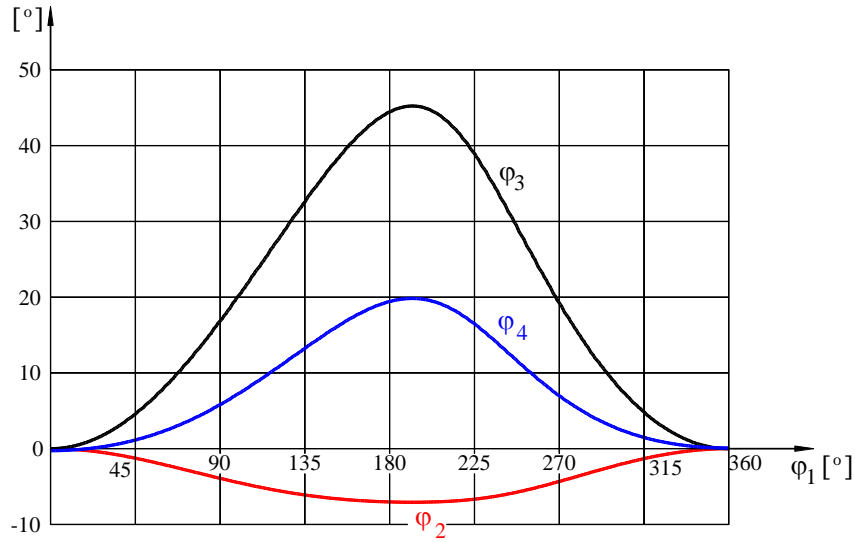


Figure 3. The variation graphs of the angles $\varphi_3(\varphi_1)$, $\varphi_2(\varphi_1)$, $\varphi_4(\varphi_1)$.

The position of the points A , B , M and D in the general reference system is given by the relations:

$$\begin{aligned} x_A &= l_{OA} \cos(\varphi_3 + \varphi_{30}), \quad y_A = l_{OA} \sin(\varphi_3 + \varphi_{30}); \\ x_B &= x_A + l_{AB} \cos(\varphi_2 + \varphi_{20}), \quad y_B = y_A + l_{AB} \sin(\varphi_2 + \varphi_{20}), \\ x_M &= x_A + x_2 \cos(\varphi_2 + \varphi_{20}) - y_2 \sin(\varphi_2 + \varphi_{20}), \quad y_M = y_A + x_2 \sin(\varphi_2 + \varphi_{20}) + y_2 \cos(\varphi_2 + \varphi_{20}); \\ x_D &= x_A + l_{AD} \cos(\varphi_2 + \varphi_{20}), \quad y_D = y_A + l_{AD} \sin(\varphi_2 + \varphi_{20}), \end{aligned} \quad (18)$$

and the expressions of angular velocity are:

$$\omega_2 = \frac{d\varphi_2}{d\varphi_1} \omega_1, \quad \omega_4 = \frac{d\varphi_4}{d\varphi_1} \omega_1. \quad (19)$$

In the paper [9] the expressions of the velocities of points A and M were established in matrix form. In the case of point M , the expression of the relative velocity v_r was also established.

The calculation program is completed with the calculation relations presented in [9] for determining the absolute velocities and the relative velocities.

The quantities obtained are stored in the form of 360-position vectors.

4. The dynamic analysis of the mechanism

In the paper [9], applying the method of isolating bodies, from 5 equations of dynamic equilibrium a linear system of 5 equations was obtained, with the solution:

$$\{R\} = [A]^{-1} \{M\} \quad (20)$$

In the relation (20) the vector of unknowns was noted with $\{R\}$:

$$\{R\} = (R_{A_x} \quad R_{A_y} \quad R_{B_x} \quad R_{B_y} \quad N)^T, \quad (21)$$

and $[A]$ is the matrix:

$$[A] = \begin{bmatrix} y_A & -x_A & 0 & 0 & 0 \\ 0 & 0 & y_B - y_C & x_C - x_B & 0 \\ \cos(\varphi_2 + \varphi_{20}) & \sin(\varphi_2 + \varphi_{20}) & \cos(\varphi_2 + \varphi_{20}) & \sin(\varphi_2 + \varphi_{20}) & -\mu \cos \beta - \sin \beta \\ -\sin(\varphi_2 + \varphi_{20}) & \cos(\varphi_2 + \varphi_{20}) & -\sin(\varphi_2 + \varphi_{20}) & \cos(\varphi_2 + \varphi_{20}) & \cos \beta - \mu \sin \beta \\ y_B - y_A & -(x_B - x_A) & 0 & 0 & a_{55} \end{bmatrix}, \quad (21)$$

$$\text{with } a_{55} = (\mu \sin \beta - \cos \beta)(l_{AB} - x_2) - (\sin \beta + \mu \cos \beta)$$

The angle β is the angle made by the friction force with the line parallel with BC in the point M where the cam and follower are in contact. The vector of unknowns $\{M\}$ is given by the expression:

$$\{M\} = \begin{bmatrix} G_3(x_{c_3} - x_O) - M_{red_3} - M_{i_3} \\ -G_4(x_C - x_{c_4}) - M_{i_4} \\ -(F_{i_{2x}} + F_{D_x})\cos(\varphi_2 + \varphi_{20}) - (F_{i_{2y}} + F_{D_y})\sin(\varphi_2 + \varphi_{20}) \\ (F_{i_{2x}} + F_{D_x})\sin(\varphi_2 + \varphi_{20}) - (F_{i_{2y}} + F_{D_y} - G_2)\cos(\varphi_2 + \varphi_{20}) \\ -M_{i_2} - F_{i_{2x}}(y_B - y_{c_2}) - (G_2 - F_{i_{2y}})(x_B - x_{c_2}) \end{bmatrix} \quad (22)$$

The masses of the following elements are considered known for the numerical application: $m_1 = 0,2$ Kg, $m_2 = 0,1$ Kg, $m_3 = 0,03$ Kg, $m_4 = 0,05$ Kg, the angular velocity of the cam $\omega_1 = 100$ rad/s, the spring 5 elastic constant k is, $k = 500$ N/m and his uncompressed length $l_0 = 0,05$ m, the sliding friction coefficient $\mu = 0,1$.

The previous calculation program is completed with the calculation relations for determining the torsion of the inertial forces acting on the elements of the mechanism, the calculation relation of the reduced moment M_{red_3} , the relations (21), (22), (23) for determining the matrix $[A]$ and of the vector $\{M\}$.

The linear system is solved, obtaining the values of the reactions: R_{A_x} , R_{A_y} , R_{B_x} , R_{B_y} and N . The values are written to value files. In fig. 4 shows the variation of the reaction from the contact point M of the cam-follower, depending on the angle φ_1 .

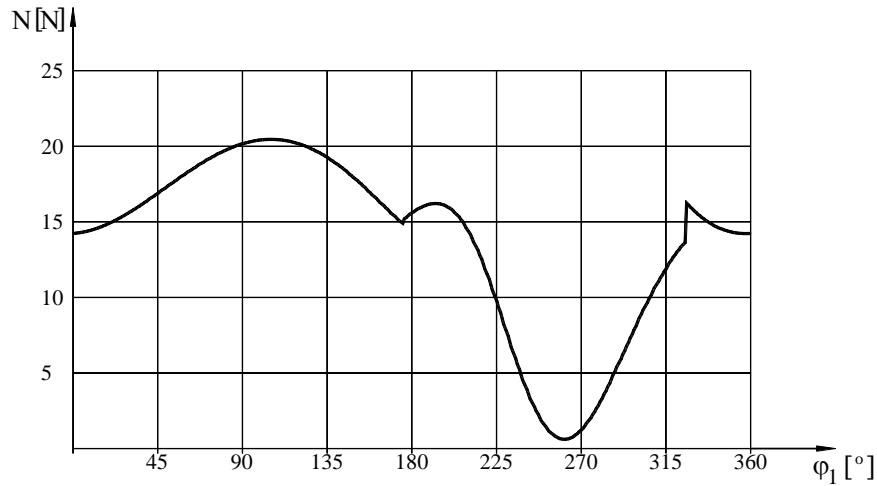


Figure 4. Variation graph of the normal reaction $N(\varphi_1)$.

5. Calculating the mechanical friction work from the cam-follower couple

In order to obtain the mechanical friction work, the value of the relative speed v_{21} from point M is determined for each position of the mechanism, which according to [9] is given by the relation:

$$\{v_{21}\} = \begin{bmatrix} v_{21x} \\ v_{21y} \end{bmatrix} = \{v_{M_6M_1}\} - \{v_{M_6M_2}\}, \quad (23)$$

and the relative velocity modulus is:

$$v_{21} = \sqrt{(v_{21x})^2 + (v_{21y})^2} \quad (24)$$

Since the values obtained for the normal reaction N and for the relative speed were obtained numerically, with a constant step of 1° , we can calculate the integral of the mechanical work with the relation:

$$L = \frac{\mu}{\omega_1} \frac{\pi}{180} \sum_{i=1}^{360} N_i |v_{21}|_i \quad (25)$$

The calculation program made previously is completed with the relations (23) ÷ (25) and the value of the mechanical friction work on a cycle is obtained at the end: $L = 0,1048 \text{ J}$.

6. Conclusions

The numerical value of the mechanical friction work in the cam-follower couple is an important criterion in assessing the efficiency of the mechanism.

The calculation algorithm presented in the paper for obtaining the value of the mechanical friction work from the cam-follower couple of a two-contour mechanism, can also be applied in the case of three-contour mechanisms or in the case of a simple cam-follower mechanism.

The numerical application follows the calculation algorithm established in a previous paper. The connecting rod of an articulated quadrilateral mechanism is driven by a circular cam. The dimensions and masses of the mechanism elements, the cam and follower profiles and the position of the spring providing the cam-follower contact are known.

The results obtained on the basis of a calculation program are presented in the form of diagrams depending on the position of the motor element. The data is stored as vectors and written to files. Finally, the value of the mechanical friction work on a dynamic cycle is obtained.

The calculation program was also used to choose the position of the spring and the value of its elastic constant, so that the value of the normal reaction is always positive, to prevent detachments.

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